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An Equilibrium Model of 'Global Imbalances' Revisited

Finn Marten Körner *

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Abstract

'Global imbalances' are almost universally regarded as a disequilibrium phenomenon. Caballero, Farhi, and Gourinchas (2008) challenge this notion with their dynamic general equilibrium model of global imbalances. The authors conclude that current account deficit nations need not worry about long-lasting deficits as long as the model is in equilibrium. The joint model in this paper combines the two model extensions for exchange rates and FDI which are disjunct in the original model. An analytical solution to the new joint model is neither as straightforward as for the separate models nor can previous results from calibrated simulation be confirmed without restriction. The model is highly dependent on parameter assumptions: A variation of calibrated parameters highlights the prime impact of investment costs previously assumed away. Sustainable equilibrium paths for global imbalances are much narrower in updated simulations than previously predicted. Policy recommendations on the sustainability of international debt holdings therefore need to be a lot more cautious.

JEL classification: F31, F34, G15, O41

Keywords: international debt, financial market development, foreign direct investment, real exchange rate, international macro-finance

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1. Introduction

Global imbalances are almost universally regarded as a disequilibrium phenomenon. Large and persistent current account balances, towering international capital flows and piling international reserves bear witness to the development of global imbalances over the course of the past two decades. Capturing these characteristics in an equilibrium model is the quest which Caballero, Farhi, and Gourinchas (2008) set out upon in the vein of the literature on international macro-finance. Their 'equilibrium model of global imbalances' strives after explaining the decline in world interest rates, the rising and persistent current account imbalances and the increase in the US share in global assets. Their model includes extensions for real-world features like exchange rates or foreign direct investment (FDI). The authors' results are clear-cut and crisp: The world ought not worry about 'global imbalances'—we are observing equilibrium phenomena. The model is in this respect a formalised sibling of the 'Bretton Woods II' hypothesis by Dooley, Folkerts-Landau, and Garber (2003, 2009).

The present paper rectifies one major caveat of the model: If exchange rates and FDI are indeed driving forces of today's globalised economy, as the authors claim, then they need to be modeled jointly. Existing side-by-side and influencing each other in reality, it is an undue oversimplification to confine them to separate sub-models. Evaluating 'global imbalances' in the light of the Caballero et al. (2008) model requires incorporating both exchange rates and FDI into one coherent framework. To the best of the author's knowledge, no attempt has been made so far to extend the equilibrium model of global imbalances both analytically and empirically. The present paper aims to fill this void left by the original model.

Analytical derivations and calibrated simulations of the joint model point to a high sensitivity to parameter assumptions. In the presence of a shock to financial markets, which is used to trigger model dynamics, a sensitivity analysis of calibrated parameters shows the tremendous impact of FDI investment costs hitherto assumed away in Caballero et al. (2008). Sustainable equilibrium paths for 'global imbalances' are much narrower than estimated previously and the benevolent conclusions from the original models cannot be drawn in the same way for the joint model. Carefully constructed estimations of domestic investment and FDI costs are a necessary prerequisite for realistic simulations. Neglecting investment costs altogether results in misleading policy recommendations. The outcome of the simulations in Caballero et al. (2008) is rather an exception than the rule in terms of total costs of investment as variations of the investment cost parameters show in simulations. More realistic assumptions for investment costs are less favourable for the investing country in the sense of restricting its ability to acquire and sustain international debt in the long-term.

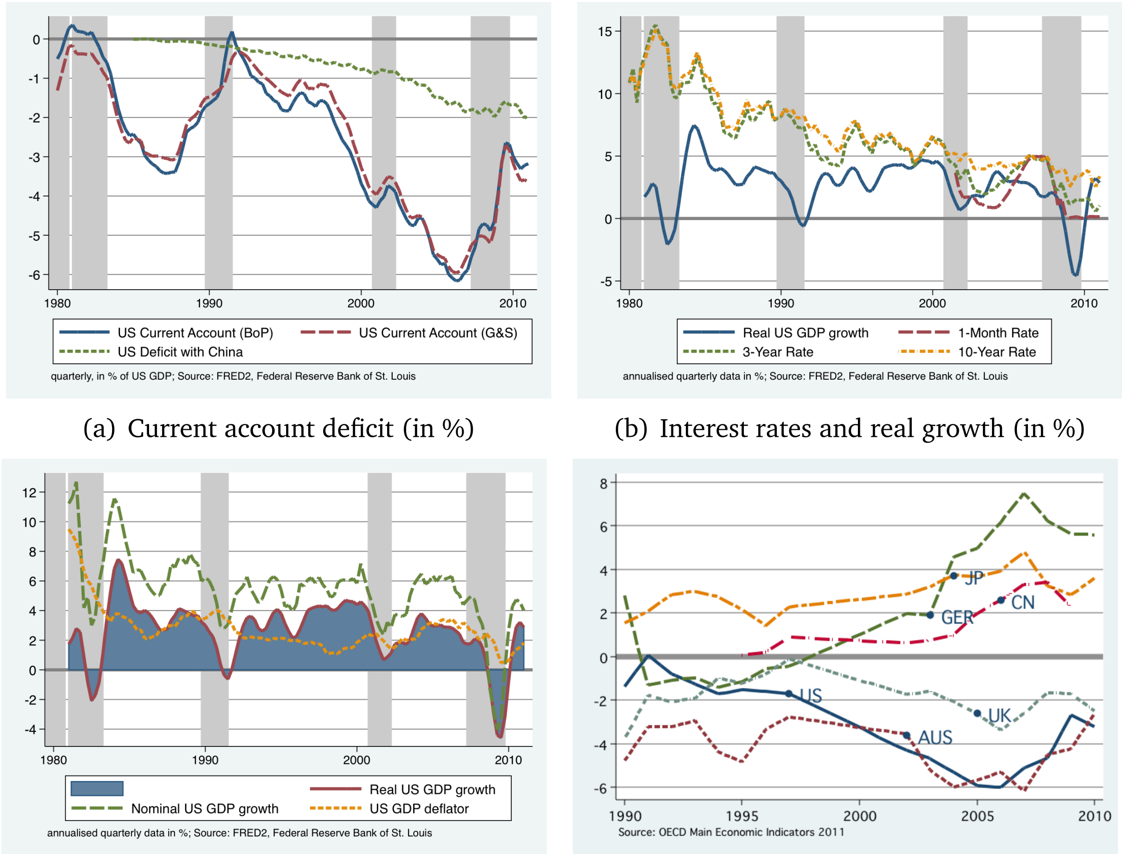
A potential further proviso lies in measuring and assessing the value of foreign assets in the investing country's wealth. The valuation of foreign assets is the international version of the issue of government bonds being 'net wealth' (Barro 1974) which recently surfaced in the discussion on a high net foreign asset position being one of strength or weakness (Wang 2007). The problem of channelling assets held abroad back home and how to deal with these assets in the long run poses a major problem to the assessment of a nation's wealth in a world of global capital markets (Cheung, Ma, and McCauley 2010). In this model, foreign assets are assumed to be fully convertible and existing foreign assets can be used to finance current account deficits. Given the many perils of international capital mobility, the present model provides the benign upper bound in this discussion while showing how more realistic scenarios look like.

The set-up of the paper is as follows. Section 2 looks at the literature on the existence and persistence of global imbalances accompanied by some empirical evidence. The two sub-models in Caballero et al. (2008) for foreign investment and exchange rates are joined together in Section 4 and their properties are derived analytically. Post-shock, dynamic and asymptotic long-term characteristics of the model are analysed and compared to the baseline model. In Section 5, calibration and simulation techniques are explained and a comparison with simulation results from Caballero et al. (2008) is made. In addition, a sensitivity analysis is presented highlighting the crucial aspects and the choice of parameters in the joint model. Lessons from the baseline model and its variations and resulting policy recommendations are drawn in the concluding section.

2. The literature on global imbalances after the crisis

The global financial crisis which erupted in mid-2007 and has been dragging on almost since then has not resulted in a serious reduction of global imbalances. Doom advocates regarded a 'dollar crash' as imminent in the run-up to the crisis, as Krugman (2007) points out. But neither a sharp decline of the external value of the US dollar nor a serious reduction of current account balances has come about as predicted by proponents of the 'end of Chimerica' (Ferguson and Schularick 2009). Global imbalances are still as 'unsustainable' as they were before the crisis in many commentators' eyes (Blanchard and Milesi-Ferretti 2009; Aizenman and Sun 2010; Corden 2011). The only notable difference is that they have been sustained for quite a long time by now having even outlasted a severe global recession. While the question whether global imbalances caused the crisis goes still unanswered (Suominen 2010), its impact on the global financial system by increasing existing tensions is largely undisputed (Claessens, Evenett, and Hoekman 2010).

Even more than before, it is now warranted to revisit the properties of 'global imbalances' and to look for alternative explanations. Caballero (2010) asks for a re-thinking of economic model-building and the way we think about macroeconomics in general. In this light, the model by Caballero et al. (2008) merits a fresh look since the baseline concept is a postulated equilibrium model of 'global imbalances'. If this is indeed the case, the global financial crisis should only have an effect on observed imbalances insofar as the underlying determinants of the model have been touched.



(a) Current account deficit (in %)

(b) Interest rates and real growth (in %)

(c) Real growth performance (in %)

(d) Global persistence in current accounts

Figure 1: Some stylised facts on global imbalances.

The tripartite pattern of global imbalances identified by, among others, Frankel (2006) and modeled by Caballero et al. (2008) is still with us today: Large and persistent current account balances, a decreasing level of real interest rates and an increase in the share of US assets in the world asset portfolio as in Figure 1. In panel (a), a notable reduction of the US current account is visible from quarterly values around 6% in 2007 to between 3% and 4% in 2010.[1] However, this represents nothing like a reversal and these values still mean significant capital imports by the US. The deficit with China, for instance, has been barely touched at all (green dashed line). Additionally, the downward trend of interest rates observed since the early 1980s has continued as in panel

[1] Current account balances using either the overall balance of payments (BOPBCA) notation or the one for goods and services (BOPBCAGS) differ only marginally as shown in Figure 1(a).

(b) despite an only modest decrease in real growth (solid blue line) as decomposed in panel (c). In addition to the FED's interest rate cuts in response to the crisis, the general decline of short-term and long-term rates visible in panel (b) appears to continue. Internationally, the picture is similar as panel (d) highlights: The global financial crisis has somewhat led to a contraction of current account balances. The general picture remains intact, though, with a distinct lack of sharp reversals for major economies and a high persistence in current account patterns.

Two questions arise from these stylised facts: If even a financial crisis and an ensuing world recession were unable to bring about a rebalancing, what does? Or, put differently, what are the underlying factors of the 'global imbalances' we are still observing? And secondly, how long is the current pattern of global imbalances sustainable and what does its sustainability depend upon?

The classical tools for an interest-rate parity analysis of current account patterns, economic growth, inflation, the interest rate and the real exchange rate, may currently be—temporarily—subdued by more finance-driven factors. The increase in capital flows, the necessary companion of ever larger trade and current account imbalances, has created its own dynamics. When elaborating on the 'future of the dollar', Cooper (2009) distinctly includes aspects like international debt and a country's overall financial position as well as long-term solvency by domestic measures. Ben Bernanke (2009) went on a similar path in his Asian perspective on the crisis speaking of the two transmission channels 'trade and finance' simultaneously. Gourinchas, Rey, and Govillot (2010) underline the Gourinchas and Rey (2007) argument that the financial role of some countries, notably the US, supersedes their former role as simple trading partners in the world economy. Linking the discussion of global imbalances with a distinct view on macro-financial aspects appears to be the right approach in light of the past crisis' inaptness to bring about real rebalancing by force.

International macro-finance is essentially a new label sticking on less rigorously spelt out theories dating back to approaches by Tobin (1969), Minsky (1982) or discussed in the Bürgenstock Papers by, e.g., Holtrop (1970) or Haberler (1970). The mathematical rigour in linking domestic macroeconomic issues to those of the international dimension has notably been intensified by Feldstein and Horioka (1980). Their seminal analysis of the puzzling link between domestic saving and international capital flows is also the basis for the present paper's model. The full-blown mathematical elaboration of today's models outshines these tentative formulations and allows a seemingly more precise analysis. Caballero et al. (2008) offer one such international macro-financial model in the tradition of Kiyotaki and Moore (1997) which we shall dwell upon.

The literature on international macro-finance is as young as it is an uncharted area.

Pavlova and Rigobon (2010) cover the multitude of models and directions this field of research has recently taken, categorising the approach followed by Caballero et al. (2008) as a current account-driven view on global imbalances with valuation effects (Pavlova and Rigobon 2010, 4). Bearing this in mind, the original model falls neatly into three sections separable by the respective rebalancing channel: (i) a baseline model using the traditional net exports channel for rebalancing, (ii) a model with investment and FDI using these proceeds to ensure the validity of the inter-temporal approach to the current account while (iii) the introduction of multiple goods produces rebalancing via the real exchange rate. All three models allow for an equilibrium outcome with infinitely lasting non-zero current account balances.

3. The model

3.1. An equilibrium model of global imbalances revisited

The model features an initially closed economy and, once opened, a large open economy as specified by Caballero et al. (2008, 7–11). The virtue of the model is its identification of imbalances in a general equilibrium framework leading to asymptotic non-zero outcomes. The focal points of this simple open-economy savings-investment model are (i) the world interest rate, (ii) current account balances and (iii) the country portfolio shares in total world assets. The model does not explicitly include a production function nor is economic growth or financial development fully endogenised, the latter two being exogenous parameters adjustable within the model. Differences in growth, reversals and counter-shocks are all modelled as ad-hoc adjustments of these parameters.

In the present set-up, the model in Caballero et al. (2008) is altered in two respects: First, the hitherto separate modelling of FDI and multiple goods with a real exchange rate is consolidated. Second, the financial development variable is partly endogenised by assuming heterogeneity among countries in terms of their capital account convertibility. Changing this assumption allows further analysis of possible tensions in three fields, namely (i) portfolio asset shares, (ii) current account imbalances and (iii) the exchange rate, all of which will be dealt with in turn. The basic model setting follows the structure of other overlapping generations (OLG) models.

Time. Time t evolves continuously.

Labour. The multitude of economic agents constitutes the labour force whose growth rate over time is assumed to be zero. Agents are born and die at the same rate θ. Population mass is constant and normalised to 1. All economy-wide variables therefore represent *per capita* values.

Production. X_t^i is the output produced at time t in region i by fully employing all available labour and capital. Output is assumed to grow at a constant endogenous rate $g^i = \dot{X}_t^i / X_t^i$ in its steady-state equilibrium. The steady-state rate of growth is equal for all variables in the economy except population growth.

Consumption path. The non-capitalisable part $1-\delta^i$ of total production X_t^i is the share of unalienable *human capital*; δ^i is the measure of a country's ability to generate financial assets from its total output and may be regarded as the degree of financial market development (Caballero et al. 2008, 363). All agents are bestowed with an initial endowment share $(1-\delta^i)$ of overall production X_t^i. At the end of their lives agents consume all their capital in its entirety, so that aggregate consumption at any time is θW_t^i. This specification is equivalent to the discounted present-value version of the OLG models in Blanchard (1985) and Weil (1989).

Savings vehicles. Over their life-time, agents employ their capital endowment in savings vehicles of identical 'trees'—an homage to the same concept in Kiyotaki and Moore (1997). Tree assets pay an aggregate dividend of $\delta^i X_t^i$ per unit time. Their value at any time t is V_t^i with capital gains of $\dot{V}_t^i / V_t^i$ per unit time.

Asset supply. The real interest rate r_t is derived from the asset supply in region i. It is the 'instantaneous return from hoarding a unit of a tree' on a micro basis. Globally, the real interest rate is equal across all regions. It serves as a vehicle for arbitrage. The following equation is determined by the supply of assets, or, if divided by V_t^i, gives the usual rate of interest r_t composed of a dividend/price ratio and the rate of change of asset prices:

$$r_t V_t^i = \delta^i X_t^i + \dot{V}_t^i \tag{1}$$

Asset demand. Agents of country i accumulate wealth W_t^i composed of the return on their savings $r_t W_t^i$, increased by the initial endowment for non-capitalisable human capital (birth contributions) $+(1-\delta^i)X_t^i$ and diminished by consumption (death withdrawals) $-\theta W_t^i$. The equation determines the change in asset demand:

$$\dot{W}_t^i = r_t W_t^i + (1-\delta^i)X_t^i - \theta W_t^i \tag{2}$$

Wealth at time t is the stock of savings W_t^i resulting from all past flows of savings $\dot{W}_t^i$. It can be re-written using the market clearing condition $\sum_i W_t^i = W_t = X_t/\theta$. The respective equilibrium growth rates for output, asset demand and asset supply are all equal and they are written in short-hand as $g = \dot{X}_t/X_t = \dot{W}_t/W_t = \dot{V}_t/V_t$. The asset market clearing condition, $W_t = V_t = X_t/\theta$, can be substituted into the above interest rate equation yielding the autonomous rate of interest in the steady state, r_{aut}, which will serve as the reference rate of interest in all sub-models:

$$\begin{aligned} r_t &= \delta X_t/V_t + \dot{V}_t/V_t \\ r_{aut} &= \delta\theta + g \end{aligned} \tag{3}$$

Equations (1) and (2) describe supply and demand in the asset market of the closed economy. This simple model of a closed economy can be opened to incorporate an open economy perspective. It then permits some heterogeneity between countries in terms of their financial market development δ^i.[2] A shock to the capitalisation rate δ^i triggers the desired model dynamics. In the baseline model, a two-country setting with regions $i = \{U, R\}$ is used. U stands for high-finance, current account deficit nations like the US, the UK and Australia while R is a label for exporting nations with a current account surplus and lesser-developed financial markets. A shock to $\delta^R < \delta$ lowers the initially identical capitalisation rates between the two regions at time $t = 0^+$. All variables denoted $t = 0^-$ are pre-shock, all afterwards are post shock. If time is denoted as $t = 0$, the variable is unaffected by the shock.

3.2. The open economy model

The closed economy in section 3.1 is opened by including trade and financial links to the rest of the world. The world is approximated by i representative economies. A trade balance (4) and a current account equation (5) are thus defined:

$$TB_t^i \equiv X_t^i - \theta W_t^i \tag{4}$$

$$CA_t^i \equiv \dot{W}_t^i - \dot{V}_t^i \tag{5}$$

It is easily discernible that these are standard national account definitions which are usually written as $NX_t = Y_t - A_t$, with A_t being domestic absorption. In this model, so far, absorption is just domestic consumption $C_t^i = \theta W_t^i$. The current account is often written, using appropriate simplifications, as domestic savings less domestic investment, $CA_t = S_t - I_t$, being the financing representation of the market for domestic assets ('trees').

The current account is the dual of the financial account (formerly capital account, cf. IMF 1993, 83). In this sense it is often helpful to think of it from an accounting perspective as the sum of the trade balance and net investment income as in (6). Any

[2] For a discussion on the link between the current account and financial market development see, among others, Caballero and Krishnamurthy (2009), Lane and Schmukler (2007) or Mendoza, Quadrini, and Ríos-Rull (2009).

country i's current account is equivalently determined by its trade balance with other countries (with $i \neq j$) while past current account balances manifest themselves in a share α_t^{ij} of country j's capital held by residents of country i. These yield asset returns or debt service depending upon the nature of cumulated past balances.

$$
\begin{aligned}
CA_t^i &= X_t^i - \theta W_t^i + r_t(\alpha_t^{ij} V_t^j - \alpha_t^{ji} V_t^i) && (6) \\
&= TB_t^i + r_t(\alpha_t^{ij} V_t^j - \alpha_t^{ji} V_t^i) && (7)
\end{aligned}
$$

The above equations are more familiar using standard notation of the current account definition including (i) net exports NX_t, (ii) net investment income $NINV_t$ and (iii) net unrequited transfers NUT_t:

$$
\begin{aligned}
S_t - I_t &\equiv CA_t && (8) \\
&= NX_t + NINV_t + NUT_t
\end{aligned}
$$

Net unrequited transfers are subsequently dropped from the current account. They are often driven by one-time political acts—foreign aid, membership of international organisations and one-off capital transfers—which are rarely theory-derived and hence usually neglected in modelling the current account. The current account definition thus simplifies to a net export component and a component signifying income from foreign asset holdings. If the country finds itself in a net debtor position to foreign creditors, net investment income is negative translating into a net payment to foreigners since foreigners' holdings of domestic assets exceed domestic holdings on foreign assets in the latter part of (6).

Some clarification is needed on the accumulation and decumulation of net asset holdings in connection with the portfolio home-bias assumption. Caballero et al. (2008, 374) define cross-border portfolio holdings as $\mu_t^{ij} = \alpha_t^{ij} V_t^j / W_t^i$. Verbally, this is the share α_t^{ij} which country i is holding in country j's assets as a share of country i's wealth. Initially, asset supply and demand are assumed to be in equilibrium and net foreign assets to be zero so that $V_{0^-}^i = W_{0^-}^i = X_{0^-}^i / \theta$. Foreign asset holdings are the product of past and present current account balances which change portfolio shares (9) and cross-border asset holdings (10) accordingly:

$$
\begin{aligned}
\text{Foreign assets share:} \quad \alpha_t^{ij} &= \frac{W_t^i - V_t^i}{V_t^j} = \frac{\sum_{s=0}^{t-1} CA_s^i}{V_t^j} && (9) \\
\text{Change in foreign assets share:} \quad \dot{\alpha}_t^{ij} &= \frac{\dot{W}_t^i - \dot{V}_t^i}{V_t^j} = \frac{CA_t^i}{V_t^j} \\
\text{Share of global portfolio:} \quad \mu_t^{ij} &= \alpha_t^{ij} \frac{V_t^j}{W_t^i} = \frac{\sum_{s=0}^{t-1} CA_s^i}{V_t^j} \frac{V_t^j}{W_t^i} = \frac{\sum_{s=0}^{t-1} CA_s^i}{W_t^i} && (10)
\end{aligned}
$$

In the calibrated simulations, initial portfolio shares and cross-border asset holdings are assumed to have a non-zero starting value, that is the sum of past current account balances is positive. Nonetheless, the current account is initially balanced and the model is in equilibrium. It is not until the shock to the capitalisation rate δ^R produces a valuation effect in addition to dynamic wealth effects that the current account starts to deviate from zero. In the presence of portfolio home-bias, existing net foreign assets in both countries are therefore run down first once the country's current account turns negative. Only when foreign assets are exhausted (i.e. $\alpha_t^{ij} = 0$) do current account deficits lead to a build-up of domestic assets by foreigners—the dual of the financial account.

The theoretical exposition uses no initial asset holdings in order not to complicate the results whereas the simulations do use small, calibrated cross-border asset holdings. Post-shock ($t = 0^+$) changes in local assets are therefore not fully absorbed by a region's domestic wealth. Instead, they are weighted according to the respective initial net asset positions of the regions at the time of the shock ($t = 0^-$):

$$W_{0+}^R = \alpha_0^{RU} V_{0+}^U + (1 - \alpha_0^{UR}) V_{0+}^R \tag{11}$$

$$W_{0+}^U = \alpha_0^{UR} V_{0+}^R + (1 - \alpha_0^{RU}) V_{0+}^U \tag{12}$$

The main channel for absorbing the initial shock to financial market capabilities in R is by the interest rate and the trade balance. The interest rate initially and permanently settles to a new equilibrium value which is lower than the autarky rate. A lower interest rate lowers wealth dynamics in U relatively more than in R and increases asset accumulation accordingly. The trade balance in U asymptotically turns into surplus while debt service makes net foreign assets and the global portfolio share converge to a constant level of international indebtedness. The main rebalancing channel is via an initial change of asset values V_{0+}^i and of the world real interest rate r_t:

$$W_{0+}^i = \alpha_0^{ij} V_{0+}^j + (1 - \alpha_0^{ji}) V_{0+}^i \tag{13}$$

$$r_t = g + (\delta - (\delta - \delta^R) x^R)\theta < r_{aut} = g + \delta\theta \tag{14}$$

$$\dot{w}_t = (r_t - \theta - g) w_t + (1 - \delta) \tag{15}$$

Caballero et al. (2008) extend their baseline model by introducing investment and a mechanism for FDI via an investment margin. We will elaborate on this in the following section.

3.3. Model extension I: Including FDI and an investment margin

The baseline model can be extended to incorporate both domestic and foreign investments. Caballero et al. (2008, 377) split output into a scale and a productivity component $X_t^i = N_t^i Z_t^i$. Using logs-and-derivatives the growth rate becomes $g = g^n + g^z$. An amount $g^n N_t^i$ of domestic investment opportunities arises continuously which requires investing a share $I_t^i = \kappa X_t^i$ of output in each period in order to attain the full potential growth rate of output. The expenditure is needed in order not to fall behind since otherwise the economy forgoes growth of g^n and only grows at a lower rate $g^z < g$. Investment may be carried out inside the economy, if investment costs κ are low enough. Investment decreases domestic asset supply (16) and attenuates wealth dynamics (17) by an increase in domestic absorption from consumption only θW_t^i to include investment $A = \theta W_t^i + I_t^i$:

$$\begin{aligned} r_t V_t^i &= \delta^i X_t^i + \dot{V}_t^i - g^n V_t^i & (16) \\ \dot{W}_t^i &= (r_t - \theta) W_t^i + (1 - \delta^i) X_t^i + g^n V_t^i - I_t & (17) \end{aligned}$$

In equilibrium, aggregate wealth and asset values balance at a new level $W_t = V_t = (1-\kappa)X_t/\theta$ depending on investment costs κ. The new equilibrium interest rate is lower than the initial autarky rate because investment increases the cost of supplying assets and decreases their value with a larger share $x^R = X_t^R / X_t$, the relative weight of country R in the global economy:

$$r_t = g^z + \frac{\theta}{1-\kappa}\left(\delta - (\delta - \delta^R) x_t^R\right) < r_{aut}^U = g^z + \frac{\delta\theta}{1-\kappa} \tag{18}$$

The investment part of the model is further extended by the opportunity for foreigners to invest in a country with lower financial market capabilities. For the purpose of exposition, the capitalisation rate in the rest of the world (R) is assumed to be inferior to U's after a shock to initially symmetric financial markets so that $\delta^R < \delta$. New investment opportunities may thence be sold by R residents to U investors. A bargaining process settles the price for new investment in R at $P_t = \kappa_P X_t^{Rn}$ assuming bilateral private gains from FDI. The cost of investment and bargaining lies between the cost for solely investing domestically in U and R respectively:

$$g^n \frac{\delta}{r_{aut} - g^z} > \kappa + \kappa_P > g^n \frac{\delta^R}{r_{aut} - g^z} \tag{19}$$

It is profitable for R to sell all new investment opportunities to U so that FDI accounts for all investment in R henceforth. In spite of this extreme assumption, it asymptotically leads to the autonomous interest rate on the right hand side of (18) which is economically optimal. The bargaining process allows both U and R residents to agree on a mutually beneficial price in line with (19) while U is also able to reap dynamic gains from its foreign investment via interest payments.

Net investment income from FDI is paramount to the equilibrium consideration of the model with FDI component and we will again take up this issue in the analysis section. Caballero et al. (2008) are able to show that a financial market shock in R does not lead to a perpetuated situation of indebtedness of U towards R. By virtue of FDI and a re-defined trade balance incorporating investment income, U is able to reduce its net foreign assets towards zero despite having both its current account and its trade balance asymptotically in deficit. The difference is accounted for by net investment income from U's FDI activity following the decrease in R's capitalisation rate.

Rebalancing in the investment extension of the baseline model takes place via a two-step process. Initially, current account deficits by U are required to sustain investment in R after the slump in its financial market. FDI activity lowers the interest rate in (20) even more than in the baseline model (14). The traditional trade balance $TB_t^i = X_t^i - \theta W_t^i - I_t^i$ is lower—and, in fact, permanently in deficit—since positive returns on U's FDI compensate for traditional trade deficits in a so-called 'non-traditional' trade balance (21). The return on FDI need to be incorporated in the current account (22) in any case feeding into wealth dynamics as in (23):

$$\begin{aligned}
r_t &= g^z + \frac{\theta}{1-\kappa}\left(\left[\delta - (\delta - \delta^R)x_t^{Ro}\right] - g^n \frac{N_0^R v_t^{Ro}}{X_t}\left[\frac{\delta}{\delta^R} - 1\right]\right) && (20)\\
\widehat{TB}_t^U &= TB_t^U + g^n V_t^R - (\kappa + \kappa_P)X_t^{Rn} && (21)\\
CA_t^U &= TB_t^U + r_t(W_t^U - V_t^U) && (22)\\
\dot{w}_t &= (r_t - \theta - g)w_t + \left(1 - \delta - \frac{\kappa}{x_t}\right) + g^n\left(\frac{1-\kappa}{\theta}\frac{1}{x_t} - \hat{v}_t^{Ro}\frac{x_t^{Ro}}{x_t}\left(1 - \frac{\delta}{\delta^R}\right) - \kappa_P\frac{x_t^{Rn}}{x_t}\right) && (23)
\end{aligned}$$

We may now turn to a separate discussion of (real) exchange rates before combining the two concepts.

3.4. Model extension II: Multiple goods and a real exchange rate

A large part of the discussion on international trade evolves around one of its most visible aspects: exchange rates. Building an exchange rate component into the equilibrium model of 'global imbalances' is thus a necessary and sensible choice. Due to the model's micro-economic structure based on an OLG framework, it is feasible to include multiple goods with a relative demand function and differentiated preferences into it. By assumption, each country produces a single good which may more realistically be imagined as an invididual country's basket of goods. A constant elasticity of substitution (CES) consumer preference function for consumption $C^i(\cdot)$ is thus defined:

$$C^i = \left(\sum_j \gamma_{ij}^{1/\sigma} x^{j\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{24}$$

with σ being the CES parameter for goods from two countries i and j while the coefficient γ_{ij} indicates the strength of preferences for the respective country's good. As is common in international trade theory, a consumption home-bias is assumed with $\gamma_{ii} \equiv \gamma > 0.5$. Fisher-ideal price indices are derived from the consumption equation above by including the regions' terms of trade, q^j, as the real demand relation between goods from i and goods from j:

$$P^i = \left(\sum_j \gamma_{ij} q^{j(1-\sigma)} \right)^{1/(1-\sigma)} \tag{25}$$

The real exchange rate between two countries i and k with respect to good j—the good produced in country j—is given by $\lambda^{ik} = P^k/P^i$. In the simple two-country case with regions U and R, we can set the price of U's good as numéraire. The real exchange rate of country U simplifies to $\lambda^{UR} = P^R/P^U$. The relative demand by inhabitants of country i for country j's good may be written as:

$$x^{ij} = \gamma_{ij} C^i \left(\frac{q^j}{P^i} \right)^{-\sigma}, \ \forall i, j \tag{26}$$

In equilibrium, the market clearing condition imposes $\sum_i x^{ij} = X^j$ for good j. As before, a country's production represents a fraction θ of the country's wealth W_t^i. Goods markets clear in equilibrium implying $P^i C^i = \theta W^i$ and wealth now being measured in terms of U's good. Good i's market clearing condition may be written using equations (24), (26) and the wealth-output-relation as in (27):

$$\theta \sum_i \gamma_{ij} \frac{W^i}{P^i} \left(\frac{q^j}{P^i} \right)^{-\sigma} = X^j, \ \forall j. \tag{27}$$

The introduction of multiple goods and a real exchange rate permits the analysis of changes in the capitalisation rate with respect to a country's exchange rate. The CES parameter σ plays a crucial role in this analysis as it accounts for the speed of adaptation to a change in circumstances by influencing the respective weights of individual countries' relative demand for goods. Relative demand between goods produced in U and R, the two-country case, are thus interpretable from an exchange rate perspective (28). The exchange rate's behaviour over time can be illustrated by taking logs-and-derivates of the output relation. Equation (28) illustrates the impact of the difference in growth rates weighted by the CES adjustment parameter on the rate of change of the terms of trade:

$$
\frac{x^{iR}}{x^{iU}} \propto \left(q^R\right)^{-\sigma}
$$
$$
\lim_{t\to\infty} \frac{\dot{q}_t^R}{q_t^R} = \frac{1}{\sigma}\left(g - g^R\right) \tag{28}
$$

In the baseline model set-up, a drop in R's capitalisation rate to $\delta^R < \delta$ does not directly affect its growth rate which remains $g^R = g$. This assumption is upheld for illustrative purposes alone while not changing the core mechanism of the model. The adjustment mechanism runs via the interest rate, as in the baseline model in 3.2, and additionally via the terms of trade. The drop in relative demand in R following the decline in its asset values after the shock to δ^R delivers a worsening of its terms of trade $q_{0+}^R < q_{0-}^R$ and therefore an initial appreciation of the exchange rate from U's viewpoint, $\lambda_{0+}^{UR} < \lambda_{0-}^{UR}$. Rebalancing in the exchange rate extension to the baseline model is thus again a two-step process: The slump in the real interest rate is more nuanced in (29) than in the baseline model (14) which is achieved by an initial real appreciation of the exchange rate from U's perspective. Consequently, wealth dynamics for $\dot{w}_t = \dot{W}_t/\dot{X}_t$ as laid out in (30) are favourable to U after the shock since a gradual depreciation attenuates the growth of its burden of international debt:

$$
\begin{aligned}
r_t &= g + \theta\left(\delta - (\delta - \delta^R)x_t^R\right) + x_t^R\left(\frac{\dot{q}_t^R}{q_t^R}\right) && (29)\\
\dot{w}_t &= (r_t - \theta - g)w_t + (1-\delta) && (30)
\end{aligned}
$$

So far, we have seen three rebalancing channels: the trade account in the baseline model in 3.1 and 3.2, FDI in the investment model extension in 3.3 and the exchange rate in the multiple goods model in 3.4. The next logical step is to combine the FDI and exchange rate components in a new model and evaluate the respective strengths of the rebalancing channels in each of them.

4. The joint model

4.1. Preliminaries

The joint model of FDI and exchange rates is a straightforward extension to the baseline scenarios considered by Caballero et al. (2006, 2008).[3] Despite coming up with a

[3] Caballero et al. (2006) is a more elaborate working paper version of the model under consideration. Model extensions include a $U-E$ comparison of an exogenous growth slump in a region comprising Europa and Japan (E) in addition to the group of current account deficit nations U (the US, the UK

large number of model variations, the authors refrain from either solving or simulating a joint model of FDI and exchange rates. Embedding this extension pushes the baseline model closer to the frontier of reality. The increased degree of complexity of the joint model alongside a decrease in clear-cut results is the natural downside of this modelling approach. It shall be pursued in the following nonetheless by first deriving model dynamics in the goods market, the asset market and its open economy properties before focusing attention on asymptotic values, convergence properties and a formulation as a non-linear dynamic system. The model uses a two-region set-up of representative countries $\{U,R\}$ with an asset and a goods market each. Country R's asset market experiences a shock to its financial market capabilities parameter δ^R which leads to the emergence of global imbalances to be studied in the model.

4.2. Model dynamics

4.2.1. The goods market

A combination of the two main extensions to the equilibrium model for FDI and exchange rates is possible since they are nested within the baseline model. The inclusion of multiple goods in the exchange rate extension affects mainly the demand side of the goods market through individual preferences and rates of substitution. The terms of trade q_t^R are determined using the equilibrium on the goods market by imposing the sum of relative demands x_t^{ij} to equal aggregate output X_t^j:

$$\sum_i x_t^{ij} = \sum_i \gamma^{ij} C_t^i \left(\frac{q_t^j}{P_t^i}\right)^{-\sigma} = X_t^j \;, \forall j \in \{U,R\}$$

Each country j's production of a representative good $X_t^j = N_t^j Z_t^j$ evolves according to the exogenous rate of growth for the number of assets ('trees') and their productivity respectively: $\dot{X}_t^j / X_t^j \equiv g = g^n + g^z$. By convention, we set the terms of trade for country U to $q_t^U = 1$ as numéraire. Total output is split into old and new trees, X_t^{Ro} and X_t^{Rn}, relating to output before and after the shock to country R's financial market parameter δ^R respectively. World output X_t can be aggregated in U's currency after being converted using R's terms of trade q_t^R:

$$X_t = X_t^U + q_t^R (X_t^{Ro} + X_t^{Rn})$$

and Australia). It also analyses a three-region set-up with U, E, and R (Asian and non-oil current account surplus nations) in a combination of effects of a growth slump in E and a financial market shock in R. The main message of the two-country model presented above remains unchanged though. In Caballero et al. (2008), extensions to incorporate ad-hoc changes to growth rates and the financial market development parameter are made without fully endogenising either. Yet, even the extended paper does not jointly model FDI and exchange rates.

The real exchange rate is thus determined as the equilibrating factor on the goods market which is required to bring about an equilibrium, given growth and investment in each of the regions respectively.

4.2.2. The asset market

Asset value and wealth dynamics equations include both investment and exchange rate characteristics. Asset supply V_t^i is determined by capital gains $\dot{V}_t^i$, financial market development δ^i, the world interest rate r_t and the growth of the number of assets $g^n V_t^i$. Asset demand change $\dot{W}_t^i$ is additionally affected by the wealth-output-parameter θ, the country's accumulated wealth W_t^i and the cost of domestic investment $I_t^i = \kappa q_t^i X_t^i$. Together, the equations define the asset market of each country:

$$\begin{aligned} \text{Asset supply:} \quad r_t V_t^i &= \delta^i q_t^i X_t^i + \dot{V}_t^i - g^n V_t^i && (31) \\ \text{Asset demand:} \quad \dot{W}_t^i &= (r_t - \theta) W_t^i + (1 - \delta^i) q_t^i X_t^i + g^n V_t^i - I_t^i && (32) \end{aligned}$$

All new investment $g^n V_t^R$ after the financial market shock to the capitalisation rate δ^R is carried out by U investors. This investment in R is profitable if the total costs of investing abroad are smaller than the financial 'know how' premium of U from $\delta > \delta^R$. The necessary condition for bilateral private gains from trade in the joint model is given by equation (33):

$$g^n \frac{\delta}{r_{aut} - g^z} > \kappa + \kappa_P > g^n \frac{\delta^R}{r_{aut} - g^z} \tag{33}$$

Investors from U buy all newly arising investment opportunities from R residents at a price P_t (34). Total investment costs are then composed of U's cost of domestic investment (35) and additionally paying an exchange rate dependent FDI bargaining price P_t:

$$\begin{aligned} P_t &= \kappa_P q_t^R X_t^{Rn} && (34) \\ I_t^i &= \kappa q_t^i X_t^i && (35) \end{aligned}$$

In this set-up with condition (33) met, carrying out FDI by U is a mutually beneficial process for both countries. Investing abroad translates for U into exporting know-how as a good which can be regarded as 'non-traditional net exports' (Caballero et al. 2008, 380). Intermediation rents from investing in foreign assets yield a higher return than the native ones abroad ($\delta > \delta^R$). Returns on FDI consequently replace U's need to run a positive trade balance with traditional net exports. The assumption of all investment in R being carried out by U through FDI grossly exaggerates the international dimension of

FDI in overall investment. However, it effectively places an upper bound to the potential explanatory power of FDI in international capital flows in this model. It takes place as long as investment abroad is profitable:

$$g^n V_t^R > (\kappa + \kappa_P) q_t^R X_t^{Rn} \tag{36}$$

The derivation of the dynamic equations for asset supply and demand is more complex than in the separate models. In the case of FDI, the interest rate is calculated using the model property that the output share x_∞^{Ro} from old pre-shock δ^R-assets becomes negligible relative to the output share x_∞^{Rn} from new post-shock δ-assets. Asset values directly after the shock in $t = 0^+$ can then be calculated by backward integration. In contrast, the terms of trade q_t^R are calculated in the pure exchange rate model using a shooting algorithm and by integrating forward the system (w_t, x_t, q_t^i) (Caballero et al. 2008, 392). The two approaches need to be reconciled.

Analytically, the model is fully identified but lacks starting values for it to be solved dynamically. In theory, we can pin down aggregate wealth dynamics in the two regions $\{U, R\}$ set-up with FDI from U to R, i.e. all investment carried out by U:

$$\begin{aligned}
\dot{W}_t^U &= (r_t - \theta) W_t^U + (1-\delta) X_t^U + g^n V_t^U + g^n N_t^R v_t^{Rn} - P_t - I_t \\
\dot{W}_t^R &= (r_t - \theta) W_t^R + (1-\delta) q_t^R X_t^{Rn} + (1-\delta^R) q_t^R X_t^{Ro} + P_t \\
\dot{W}_t &= (r_t - \theta) W_t + (1-\delta)(X_t^U + q_t^R X_t^{Rn}) + (1-\delta^R) q_t^R X_t^{Ro} + g^n (V_t^U + N_t^R v_t^{Rn}) - I_t
\end{aligned} \tag{37}$$

The mechanism which creates the model's asymmetry is derived from FDI returns on assets. P_t is the selling price of investment opportunities in R to U. Overall wealth is not curtailed by the bargaining in this allocation process as long as FDI does take place and all investment opportunities are used. The notable difference exists between investment in R's old assets V_t^{Ro} before the shock and into new assets V_t^{Rn} thereafter. At the time of the shock at time $t = 0^+$, the number of existing assets is N_0^R while new investment up until time t is $N_t^R - N_0^R$. Investment goes only into new assets, so that any capital gain on existing assets can only be due to productivity increases yielding $\dot{V}_t^i = g^z V_t^i$ in equilibrium.

For investment in U and new investment in R, gross returns are reduced by the cost of investing into all new asset opportunities $g^n V_t^{Rn}$ opening up. The total number of assets for V_t^U and $V_t^{Rn} = (N_t^R - N_0^R) v_t^{Rn}$ increases at a rate g^n while the number of old assets $V_t^{Ro} = N_0^R v_t^{Ro}$ remains constant after the shock.[4] Domestic returns on old and new assets in R and U with full FDI satisfy the following set of equations which can be aggregated

[4] A vintage parameter accounting for asset depreciation could be introduced to accelerate the diminishing importance of old assets in R even more.

globally to give the global change in asset values (38):

$$
\begin{aligned}
r_t V_t^{Ro} &= \delta^R q_t^R X_t^{Ro} + \dot{V}_t^{Ro} \\
r_t V_t^{Rn} &= \delta q_t^R (N_t^R - N_0^R) Z_t^{Rn} + (N_t^R - N_0^R) \dot{v}_t^{Rn} \\
&= \delta q_t^R X_t^{Rn} + \dot{V}_t^{Rn} - g^n N_t^R v_t^{Rn} \\
r_t V_t^U &= \delta X_t^U + \dot{V}_t^U - g^n V_t^U \\
\dot{V}_t &= r_t V_t - \delta (X_t^U + q_t^R X_t^{Rn}) - \delta^R q_t^R X_t^{Ro} + g^n (V_t^U + N_t^R v_t^{Rn})
\end{aligned} \tag{38}
$$

Investment by U in R makes asset supply and asset allocation take separate paths in response to the financial market shock to δ^R. The divergence in asset allocation leads to a convergence in the net asset position since dividends from FDI accruing to U's wealth turn around the net debtor position which U finds itself in by accomodating the shock through lowered world interest rates and increased current account deficits.

Globally, aggregate asset demand (37) and aggregate asset supply (38) need to be balanced at all times. Inter-regional developments with divergent asset demand and supply cannot in any case lead to deviations from equilibrium in the asset market $V_t = W_t$. In addition to stocks, flows need to be globally balanced, too, in each period:

$$
\begin{aligned}
\dot{W}_t &= \dot{V}_t \\
(37) &= (38) \\
W_t &= V_t = \frac{(1-\kappa)}{\theta} X_t
\end{aligned} \tag{39}
$$

using $I_t = \kappa X_t = \kappa \left[X_t^U + q_t^R (X_t^{Ro} + X_t^{Rn}) \right]$ in the latter equation. Returns on FDI are repatriated via the capital account which may asymptotically balance persistent deficits in the current account. Adding exchange rates creates a continued slide of U's currency against R as a second rebalancing channel sometimes described as the 'slow decline of the dollar' mechanism in reality. A depreciation effectively decreases outstanding international liabilities of U to R because U's debt is in assets denominated in its own currency. The reverse case is happening with dividend payments earned by U from its FDI in R: constant returns from U investment in R translate via a depreciating exchange rate into higher net investment income in U's balance of payments. However, in order to keep investment at the same level using all investment opportunities at each point in time, an ever increasing price $P_t = \kappa_P q_t^R X_t^{Rn}$ needs to be paid by U which decreases domestic wealth. Taken together both effects create a possible dynamic inconsistency which can only be resolved by calibrated simulations; in particular, the problem requires a discussion of the critical choice of parameter values $\kappa + \kappa_P$ since they determine the long-term solvency of the investing country U.

4.2.3. Open economy

The interaction of the two rebalancing channels—net returns on FDI and the evolution of the exchange rate—has a profound effect on the countries' net asset positions. Substituting the equilibrium condition on the asset market, $\theta V_t = X_t(1-\kappa)$ from (39) into the asset supply equation (38) solves for the interest rate (40) :

$$\begin{aligned} r_t &= \frac{\theta}{(1-\kappa)X_t}\left[\delta\left(X_t^U + q_t^R X_t^{Rn}\right) + \delta^R q_t^R X_t^{Ro} + \dot{V}_t - g^n\left(V_t^U + N_t^R v_t^{Rn}\right)\right] \\ &= \dot{V}_t \frac{\theta}{(1-\kappa)X_t} + \frac{\theta}{(1-\kappa)X_t}\left[\delta\left(X_t^U + q_t^R X_t^{Rn}\right) + \delta^R q_t^R X_t^{Ro} - g^n\left(V_t^U + N_t^R v_t^{Rn}\right)\right] \end{aligned}$$

The first term can be simplified using (39). The steady-state rate of growth of all variables is $g = g^z + g^n = \dot{X}_t/X_t = \dot{V}_t/V_t = \dot{W}_t/W_t$ and can be substituted yielding:

$$\begin{aligned} r_t &= \frac{\dot{X}_t}{X_t} + \frac{\theta}{(1-\kappa)}\left[\delta\left(x_t^U + q_t^R x_t^{Rn}\right) + \delta^R q_t^R x_t^{Ro} - g^n \frac{\left(V_t^U + N_t^R v_t^{Rn}\right)}{X_t}\right] \\ &= \frac{\dot{X}_t}{X_t} - g^n + \frac{\theta}{(1-\kappa)}\left[\delta\left(x_t^U + q_t^R x_t^{Rn}\right) + \delta^R q_t^R x_t^{Ro} - g^n \frac{N_0^R\left(v_t^{Rn} - v_t^{Ro}\right)}{X_t}\right] \end{aligned} \quad (40)$$

Using $V_t = V_t^U + (N_t^R - N_0^R)v_t^{Rn} + N_0^R v_t^{Ro}$ in the last term of equation (40) yields the asymptotic interest rate for the model with FDI and exchange rates. The assumption of a uniform steady-state growth rate requires a derivation of the equation for growth measured in U's goods, for which we know (41) and (42) from the separate model extensions. We can then derive the joint model growth rate:

$$\begin{aligned} \text{FDI}: \frac{\dot{X}_t}{X_t} &= g^n + g^z = g & (41) \\ \text{XR}: \frac{\dot{X}_t}{X_t} &= g x_t^U + \left(g^R + \frac{\dot{q}_t^R}{q_t^R}\right) x_t^R & (42) \\ \text{FDI} + \text{XR}: \frac{\dot{X}_t}{X_t} &= \frac{g \sum_i q_t^i X_t^i}{X_t} = \frac{g\left(X_t^U + q_t^R X_t^R\right)}{X_t} \\ &= \frac{g\left(N_t^U Z_t^U + q_t^R N_t^R Z_t^R\right)}{X_t} \\ &= (g^n + g^z) x_t^U + \left(g^n + g^z + \frac{\dot{q}_t^R}{q_t^R}\right) x_t^R \\ &= g^n + g^z + \frac{\dot{q}_t^R}{q_t^R}(1 - x_t^U) & (43) \end{aligned}$$

The combined growth equation in (43) has ingredients from the FDI part, notably the separation into a scale and a productivity component, as well as the weighted appreciation term from the multiple goods extension. After time, the exchange rate adjustment to the shock fades out and converges to nil. This asymptotically stabilises relative output shares of the two countries x_t^i:

$$\lim_{t\to\infty} \dot{x}_t^U = x_t^U(1-x_t^U)\left(g - g^R - \frac{\dot{q}_t^R}{q_t^R}\right) = 0$$

After a shock to δ^R, but before reaching the asymptotic steady state, $\dot{q}_t^R/q_t^R$ will be different from 0: An initial decrease prompted by increased asset demand of R in U is followed by a slow and steady ascent by unwinding these positions. Substituting (43) into (40) gives the interest rate for the two regions case with FDI and exchange rate (XR):

$$r_t = g^z + \frac{\dot{q}_t^R}{q_t^R}(x_t^{Rn} + x_t^{Ro}) + \frac{\theta}{(1-\kappa)}\left[\delta - (\delta - \delta^R)x_t^{Ro} - g^n \hat{v}_t^{Ro} x_t^R \left(\frac{\delta}{\delta^R} - 1\right)\right] \tag{44}$$

The common world interest rate of the joint model is affected by an output-weighted component of R's exchange rate adjustment vis-à-vis U and another part which is affected by the capitalisation rate differences leading to FDI. Output *produced in R* therefore differs from production capacity *owned by R*. We need to look at wealth dynamics and asset values to see what this means for net foreign asset positions of the two regions in the long term.

The nature of the equilibrium model translates an equilibrium on domestic asset markets into corresponding values in the international sphere. Before the shock, all international positions are zero and the model is in equilibrium as are the trade balance and the current account of each country. From U's perspective these take the following form for the trade balance, net assets and the current account respectively:

$$TB_t^U = X_t^U - I_t^U - \theta W_t^U. \tag{45}$$

$$NA_t^U = \alpha_t^{UR} V_t^R - \alpha_t^{RU} V_t^U \equiv W_t^U - V_t^U \tag{46}$$

$$CA_t^U = TB_t^U + r_t(\alpha_t^{UR} V_t^R - \alpha_t^{RU} V_t^U) \equiv \dot{W}_t^U - \dot{V}_t^U \tag{47}$$

The structure of the above equations illustrates the way international adjustment takes place in the model. A shock to the financial market parameter δ^R increases U's asset values vis-à-vis R's lifting its wealth accordingly. While output and investment are initially unaffected by the shock, the trade balance and the current account of U react to it by going into deficit driven by a change in relative asset prices and an initial appreciation of U's exchange rate. A lower interest rate and a gradual depreciation determine the increase in debt, i.e. the amount of net foreign assets U can sustain to counter the shock. The larger the shock, the larger the counter-balancing reaction by U. Net foreign assets are the result of cumulated past current account balances. The ratio α_t^{ij} in (48) gives the share of assets of country j owned from abroad. The global portfolio share (49) is the ratio of past current account balances as a share of domestic wealth, or the share of a country's wealth which is held abroad:

$$
\begin{aligned}
\text{Foreign assets share:} \quad \alpha_t^{ij} &= \frac{W_t^i - V_t^i}{V_t^j} = \frac{\sum_{s=0}^{t-1} CA_s^i}{V_t^j} \qquad (48)\\
\text{Change in foreign assets share:} \quad \dot{\alpha}_t^{ij} &= \frac{\dot{W}_t^i - \dot{V}_t^i}{V_t^j} = \frac{CA_t^i}{V_t^j}\\
\text{Global portfolio share:} \quad \mu_t^{ij} &= \alpha_t^{ij} \frac{V_t^j}{W_t^i} = \frac{\sum_{s=0}^{t-1} CA_s^i}{V_t^j} \frac{V_t^j}{W_t^i} = \frac{\sum_{s=0}^{t-1} CA_s^i}{W_t^i} \qquad (49)
\end{aligned}
$$

The foreign asset share and the share of U assets in the global portfolio are the gauge for international indebtedness in the model. They serve as a warning light for deviations from the optimal path of net foreign assets leading away from asymptotic equilibrium.

4.3. Asymptotic values

The model converges asymptotically to its long-term position as determined by the dynamic equations above. The wealth dynamics equation (37) may be re-formulated to feed into trade balance and current account output shares as follows:

$$
\begin{aligned}
W_t^U &= \frac{(1-\delta)X_t^U + g^n V_t^U + g^n N_t^R v_t^{Rn} - \kappa_P q_t^R X_t^{Rn} - \kappa X_t}{(g^z + \theta - r_t)}\\
\frac{W_t^U}{X_t^U} &= \frac{(1-\delta) + g^n \left(V_t^U + N_t^R v_t^{Rn}\right)/X_t^U - \kappa_P x_t^{Rn}/x_t^u - \kappa/x_t^u}{(g^z + \theta - r_t)} \qquad (50)
\end{aligned}
$$

where $\lim_{t\to\infty} x_t^{Rn}/x_t^R = 1$ since output continues to grow at a rate $g^R = g$ making output from old trees, x_t^{Ro}, disappear relatively in the long-term. The trade balance is the dual of R's trade balance in a two-regions setting; it therefore suffices to look at U's asymptotic values alone. Substituting (50) into $TB_t^U \equiv -\theta W_t^U - I_t^U + X_t^U$ and dividing by X_t^U yields:

$$
\frac{TB_t^U}{X_t^U} = -\theta \frac{(1-\delta) + g^n (V_t^U + N_t^R v_t^{Rn})/X_t^U - \kappa_P x_t^{Rn}/x_t^u - \kappa/x_t^u}{(g^z + \theta - r_t)} + (1-\kappa) \qquad (51)
$$

The sign of the trade balance in the long-term is undetermined. It depends on the distinction between old and new trees, x^{Ro} and x^{Rn}, and on the evolution of the exchange rate. A priori, we cannot make a statement on whether we observe dynamic inconsistencies when a slowly depreciating currency of U increases both investment costs and investment returns at the same time. The same indeterminacy is true of the current account which satisfies asymptotically:

$$
\frac{CA_t^U}{X_t^U} = g\left(\frac{(1-\delta) + g^n (V_t^U + V_t^{Rn})/X_t^U - \kappa_P x_t^{Rn}/x_t^U - \kappa(1 + x_t^{Rn}/x_t^U)}{(g^z + \theta - r_t)} - \frac{\delta}{r_t - g^z}\right) \qquad (52)
$$

U derives positive income from FDI because the global interest rate is higher than its cost of investing abroad. This feature has long been observed of US foreign investment which tended for decades to yield a higher return than US debt payments to foreigners.[5] In the model, the relation between domestic and foreign investment costs, κ and κ_P is not predetermined. In order to make specific policy recommendations using the joint model, we need to turn to calibrated simulations.

4.4. Non-Linear System Dynamics

The above equations constitute a non-linear dynamic system which can can be split into its components. The model consists of four flow variables $(\dot{w}_t, \dot{q}_t^R, \dot{x}_t, \dot{\hat{v}}_t^{Ro})$, five state variables $(w_t, q_t^R, x_t, \hat{v}_t^{Ro}, r_t)$, an exogenous variable $(g = g^R = g^z + g^n)$, six auxiliary variables $(P_t^U, P_t^R, \lambda_t^{UR}, x_t^R, x_t^{Rn}, x_t^{Ro})$ and eight constants $(\theta, \delta, \delta^R, \kappa, g^n, g^z, \sigma, \gamma)$. The system is sequentially constructed as specified in Appendix A.1.3. Its four non-linear dynamic equations are:

$$\dot{w}_t = (r_t - \theta - g)w_t + (1 - \delta - \frac{\kappa}{x_t}) + g^n \left[\frac{(1-\kappa)}{\theta x_t} + \hat{v}_t^{Ro} \frac{x_t^{Ro}}{x_t} \left(\frac{\delta}{\delta^R} - 1 \right) \right] - \kappa_p \frac{x_t^{Rn}}{x_t} \quad (53)$$

$$1 = \theta \gamma w_t P_t^{U(\sigma-1)} + (1-\gamma) \left(\frac{(1-\kappa)}{x_t} - \theta w_t \right) P_t^{R(\sigma-1)} \quad (54)$$

$$\dot{x}_t = x_t(1-x_t) \left(g - g^R - \frac{\dot{q}_t^R}{q_t^R} \right) \quad (55)$$

$$\dot{\hat{v}}_t^{Ro} = \frac{\theta}{1-\kappa} \left[\delta(1 - x_t^{Ro}) + \delta^R x_t^{Ro} - g^n \hat{v}_t^{Ro} x_t^{Ro} (\delta/\delta^R - 1) \right] \hat{v}_t^{Ro} - \delta^R \quad (56)$$

The remaining state variable and auxiliary equations are given by:

$$r_t = g^z + x_t^R \frac{\dot{q}_t^R}{q_t^R} + \frac{\theta}{(1-\kappa)} \left[\delta - (\delta - \delta^R) x_t^{Ro} - g^n \hat{v}_t^{Ro} x_t^R \left(\frac{\delta}{\delta^R} - 1 \right) \right] \quad (57)$$

$$P_t^U = \left(\gamma + (1-\gamma) q_t^{R(\sigma-1)} \right)^{1/(1-\sigma)} \quad (58)$$

$$P_t^R = \left(\gamma q_t^{R(\sigma-1)} + (1-\gamma) \right)^{1/(1-\sigma)} \quad (59)$$

$$\lambda_t^{UR} = P_t^R / P_t^U \quad (60)$$

$$x_t^R = 1 - x_t = x_t^{Ro} + x_t^{Rn} \quad (61)$$

$$\dot{x}_t^R = -\dot{x}_t = \dot{x}_t^{Ro} + \dot{x}_t^{Rn} \quad (62)$$

[5] Gourinchas and Rey (2007) and Gourinchas, Rey, and Govillot (2010) explore the hypothesis of a "valuation channel" in contrast to the well-established "trade channel" for rebalancing. They argue that an improved return on US net foreign assets either via an accommodating return spread or via a dollar depreciation is sufficient to balance present current account imbalances and even reduce net foreign liabilities from the past.

The dynamic system cannot be written in closed form or in a matrix representation. It does, however, have full rang making it uniquely soluble with approximation methodology. We have to resort to calibrated simulations in combination with a shooting mechanism to solve for the post-shock value for $\dot{q}^R_{0^+}$ and an iterative approach to find $\hat{v}^{Ro}_{0^+}$, the asset value output share of old assets in R directly after the shock. Given these post-shock values, the system can be dynamically derived.

5. Results

5.1. Calibration

The equilibrium model of global imbalances is tested and solved in calibrated simulations. Model simulations with calibrated parameters and starting values allow a reality check whether the model captures the essential features of global imbalances as described in the beginning. The joint model is nested within the baseline model of Caballero et al. (2008) making results comparable to those from the individual models with the same model parameters. Each model can be fully embedded in the other by virtue of either blocking the FDI rebalancing channel or the one for the exchange rate.

Model simulations use calibrated starting values analogous to Caballero et al. (2008). The drop in asset values in response to the slump in the financial market parameter δ^R is calibrated to –25%—the working paper (2006) uses a 50% drop. In contrast to the theoretical exposition above, small non-zero calibrated asset cross-holdings $\mu^{RU}_{0^-}$ are used to capture the interconnectedness of asset markets and to allow for shock-driven valuation effects. All other calibrated values remain unchanged to facilitate comparison:

Parameter	θ	g	δ	x^R_0	$\mu^{RU}_{0^-}$	$NA^U_{0^-}$	σ	γ	g^z	g^n	κ	κ_P	r_{aut}
Value	0.25	0.03	0.24	0.30	0.05	0.0	4	0.9	0.0	0.03	0.0	0.12	0.06

Table 1: Originally calibrated and starting values for exogenous model parameters.

Calibrating investment costs warrants particular emphasis. Caballero et al. (2008, 380) give neither written nor numeric account of why they assume investment costs to be negligible by setting $\kappa = 0$. In the same way, the FDI bargaining parameter is arbitrarily set to $\kappa_P = .12$. In contrast to all other parameter values, which are calibrated using real-world economic data, both costs of domestic and foreign investment are lacking an empirical foundation. For reasons of comparability, we will uphold these assumptions but we will revisit the need to calibrate investment costs when conducting a sensitivity analysis in section 5.3. New values for investment costs are calibrated in

Appendix A.1.1.

5.2. Simulation

The joint model can be simulated numerically using the above calibrated values as starting points. The model is fully defined by the equations in Section 4.4 but needs to be translated to be numerically soluble using the complex structure presented in Appendix A.1.3. Initially, the model is fully solved prior to the shock at $t = 0$ in order to verify all assumptions concerning the model's stationarity (cf. A.1.4). At the time of the shock, new values need to be iteratively obtained using a shooting algorithm and iterative loops as specified in Appendix A.1.5. Given these post-shock values, the set of dynamic equations as defined in A.1.6 describes the evolution over time. The model is fully programmed in Stata using object-oriented techniques in order to be able to monitor developments at all stages of the simulation and not to face a black box of results. The model converges towards the asymptotic values derived theoretically when $t \to \infty$, here using $N = 200$ periods/years. A reduced set of results for the joint model and its variations is presented for comparison in the summary Tables A.2 through A.10 in Appendix A.2.

The simulation results in Figure 2 show the differences between the three separate models and the joint model. The behaviour of the real interest rate in the bottom left panel is indicative of the general picture: In the joint model, the interest rate falls from 6% to 5.35%. It is thus stronger than in the baseline (5.55%) and in the pure exchange rate (XR, 5.59%) model but it does not fall as deep as in the pure FDI model with 5.10% (see Table A.2 for details). Gradual replacement of old by new assets in R through FDI makes the real interest rate converge to its pre-shock equilibrium value—in contrast to the two cases without FDI in which it remains depressed.

The picture for the real exchange rate in the bottom centre panel exhibits an initially strong appreciation of U's exchange rate by 5.7%, slightly more than the one in the pure exchange rate model (4.3%). But the terms of trade are not any more the only rebalancing channel in the joint model: The presence of foreign investment makes the exchange rate absorb less of the post-shock rebalancing. Asymptotic depreciation takes place more slowly in the joint model for which the real exchange rate converges to its starting value of 1.13, while in the pure XR model it depreciates a further 3.4%. In contrast to the pure exchange rate model, we do not see the global portfolio share of U assets in R's portfolio converge to a level around 20% but to rise well above 100% of output asymptotically. The net asset output share indicates higher international indebtedness in excess of 100% of U's output in the model.

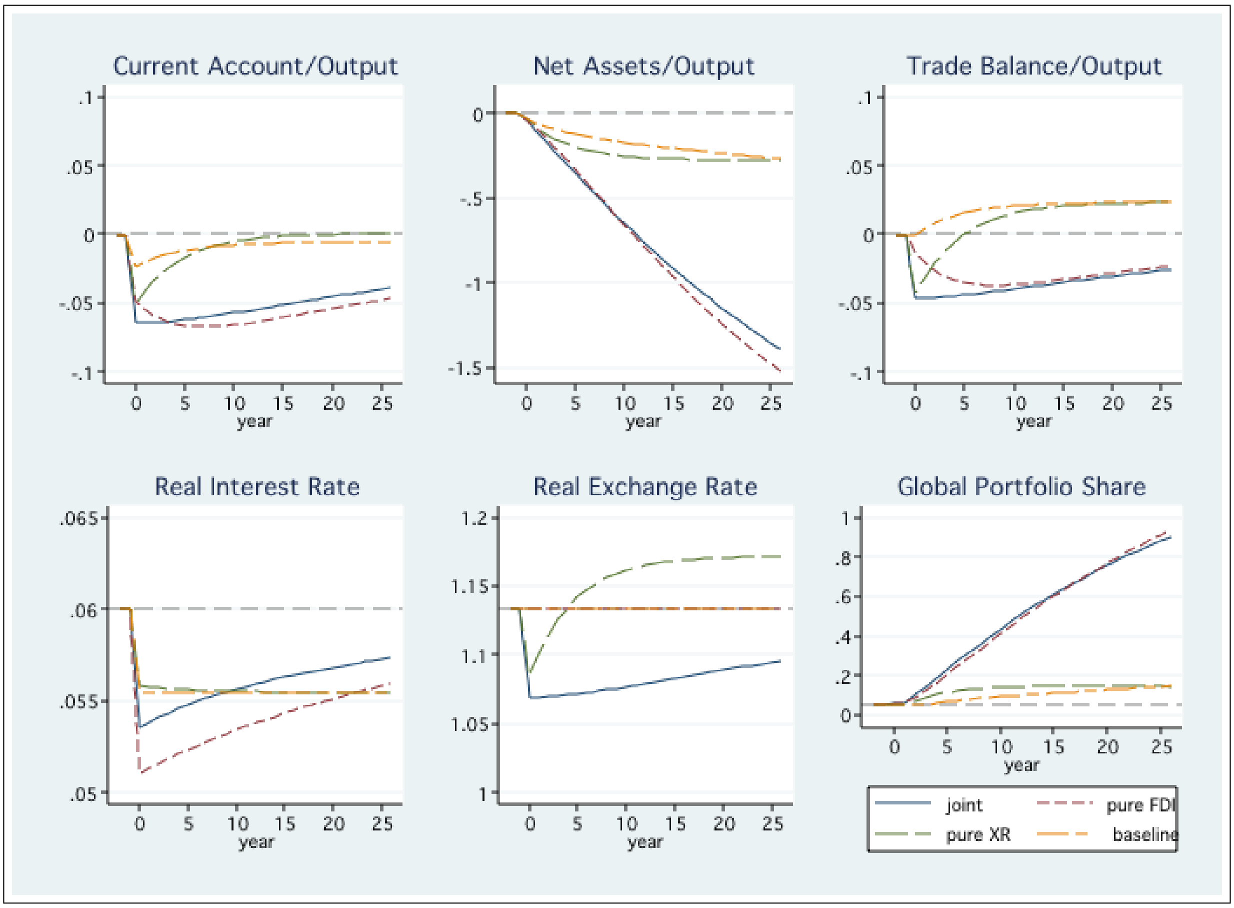


Figure 2: Baseline results from the joint model with $\delta^R = .18$ producing a 25% shock.

The initial current account reaction of the joint model is -6.45%, stronger than in either of the other sub-models. The combination of an initial appreciation and the possibility to conduct foreign direct investment allows a larger initial reaction of the current account. Likewise, we see a strong initial reaction of the trade balance which stays in deficit financed by returns on FDI as in the pure investment model.

In summary, the combination of exchange rates and FDI in Figure 2 produces similarly benevolent results as for the separate models previously. The reaction of the current account and the resulting level of international debt suggest that the joint model behaves along the same lines. In order to confirm the stability of these results, we turn to the crucial choice of model parameters and their calibration. A variation of each of the parameters and some of them jointly makes the dynamic processes better understandable. The sensitivity analysis brings to light that the benign results from the baseline joint model cannot be generally upheld when assuming more realistic parameter values.

5.3. Sensitivity analysis

5.3.1. A variation in the financial market development parameter δ^R

A variation in the financial market variable δ^R lies at the heart of the Caballero et al. (2008) equilibrium model. The parameter captures the decline in credibility of domestic assets resembling the 1998 Asian crisis. The shock in region R is aimed to produce the current account and capital account pattern of R-like countries in the real world like Asian industrial goods exporters and other non oil-exporting surplus nations. Their surplus in the current account is mirrored by an increase in their portfolio holdings of assets from deficit nations, i.e. an increase in the U region countries' international indebtedness.

The joint model is solved for different values of δ^R which produce shocks of varying magnitudes as indicated by the legend to Figure 3. Depending upon the size of the change in δ^R the shock to asset values V^R_{0+} varies between –62% ($\delta^R = .09$), –25% for the reference case of the joint model with $\delta^R = .18$, no shock for $\delta^R = .24$ and plus 12% for an increase in δ^R from .24 to .27. No matter the size of the initial shock, all asset value paths converge to a similar rate of increase with the aid of FDI from U so that output in R is not depressed forever.

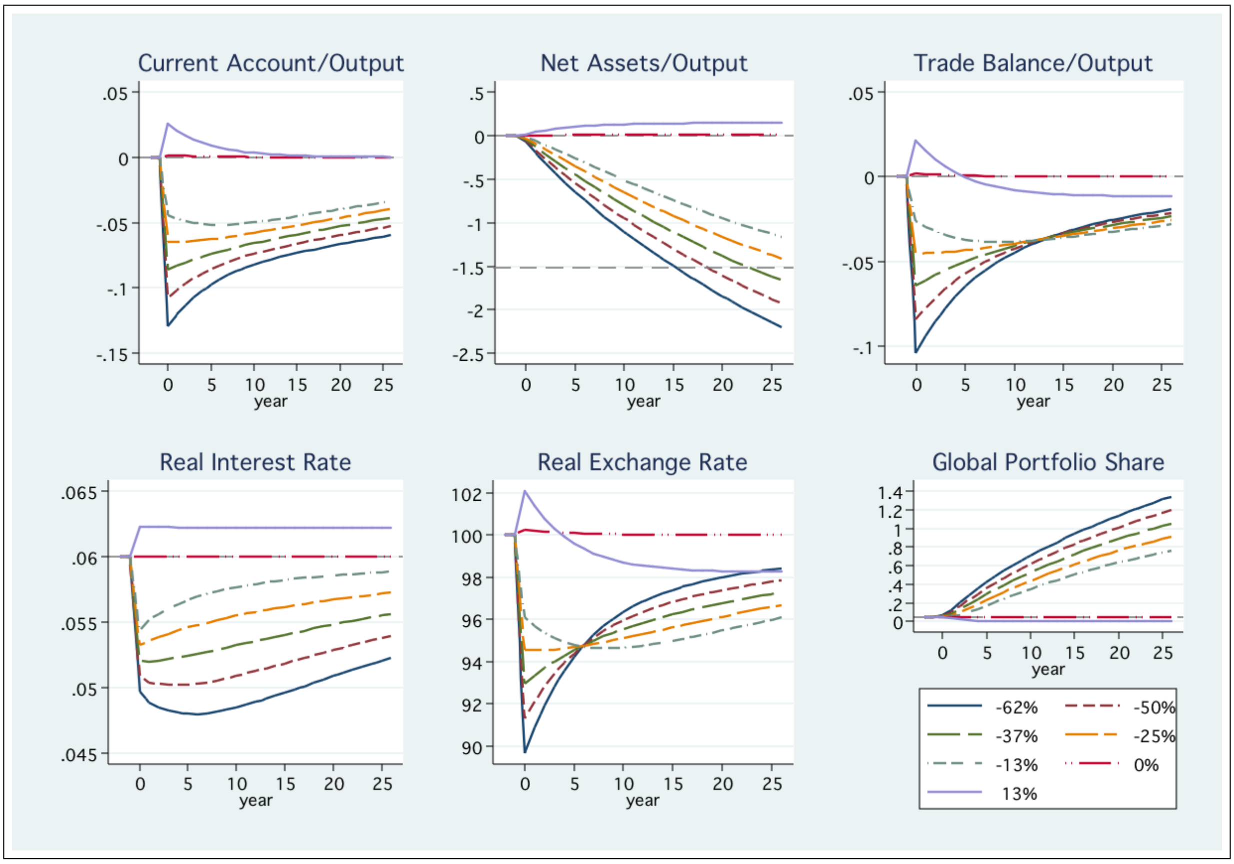


Figure 3: A variation of the financial market development parameter δ^R.

The outcomes of the variation of the δ^R parameter are similar for the interest rate and the real exchange rate (bottom left and centre panels of Figure 3, respectively). The path of the interest rate decreases at impact and slowly reverts towards its asymptotic value of $r_{aut} = r_\infty = .06$. The larger the shock to δ^R the higher the initial drop in the interest rate and the larger the initial appreciation and the faster the eventual depreciation of U's exchange rate.

The response of the current account is similarly pronounced: Again, it initially goes into deficit by an amount similar to the one in the separate models. The stronger the decrease in δ^R the lower the current account at impact. We observe persistent current account deficits in U for any value of δ^R below its initial value of .24. Net assets are deep in the red and only manage a turn-around at international debt being up to twice the size of output (top centre). Even moderate shocks lead to large negative net foreign assets of U—but these paths can be contained. The global portfolio share of U assets in R's portfolio converges at a significantly higher level than in the separate sub-models. By reverting to zero, the trade balance absorbs the rising investment costs in U and R due to the depreciation of the real exchange rate.

5.3.2. A variation in the exchange rate adjustment speed σ

The parameter defining the strength of the exchange rate rebalancing channel is the consumption preference parameter σ from the consumption and relative demand equations. The lower the parameter σ the stronger the initial reaction and the faster the adjustment of the exchange rate to changing circumstances, as shown in the bottom centre panel of Figure 4 with q^R_{0-} normalised to 100 for comparison. A stronger exchange rate response requires a weaker reaction of the interest rate as in the bottom left panel. For $\sigma = 4$ the baseline case from Figure 2 is reproduced. A stronger exchange rate reaction, i.e. a larger initial appreciation of U's terms of trade, leads to lower current account and trade balance deficits of U associated with a lower portfolio share of U assets in R's portfolio (bottom right). The level of international indebtedness as measured by net assets over output is correspondingly higher for higher values of σ.

Tweaking the exchange rate adjustment speed produces some variation in the model. Even though the resulting real exchange rate differs significantly for different values of σ, the main message from the joint model holds unreservedly. An asset market shock therefore produces the well-known ‘safe haven’ story for U's exchange rate followed by a gradual and slow decline. It is the picture we have seen after the Asian and the global financial crisis but it cannot explain the emergence of ‘global imbalances’ in the model. These imbalances must stem from the investment parameters in the model and are only facilitated by the accompanying exchange rate behaviour.

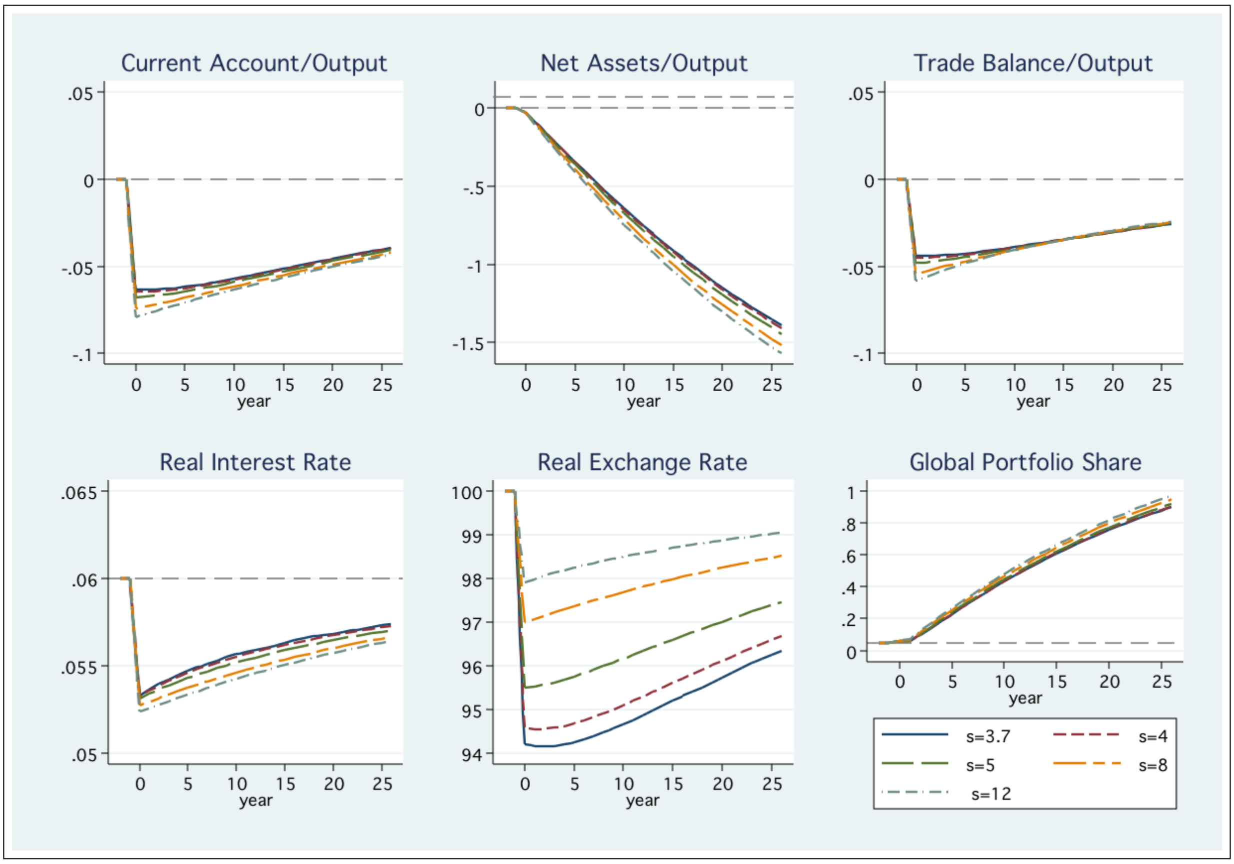


Figure 4: A variation of the terms of trade response parameter σ.

5.3.3. A variation in the investment cost parameter κ

Investment costs are the decisive parameter of the model. Caballero et al. (2008) assume them to be negligible arguing that it suffices for the sum of investment costs κ and the FDI bargaining price κ_P to be set so as to allow bilateral private gains from trade—cf. equation (33). In the simulations, though, only κ_P is varied while leaving $\kappa = 0$ at all times. A variation of κ between 0% and 15% as presented in Figure 5 is effectively a change in the net investment rate of the economy. We can see that such a variation has extremely pronounced effects for the same 25% shock to R's assets .

The larger overall investment costs, the higher the equilibrium interest rate (bottom left panel). $\kappa = 0$ simply reproduces the baseline case from the joint model. In terms of dynamics, this case is the exception rather than the rule compared with all non-zero parameter values. Bilateral private gains from trade defined in inequality (33) require a value of $\kappa + \kappa_P$ below 12% on the upper bound.[6] The cost of (domestic and foreign) investment for U is otherwise prohibitively high and needs to be countered by an even stronger interest rate and exchange rate reaction. The graphs for $\kappa = .15$ illustrate the costs to U of running persistent current account deficits in the presence of high

[6] A back-of-the-envelope calculation tells us that for $g^n\delta^i/(r_{aut} - g^z)$ the sum of investment costs and the bargaining price for FDI, $\kappa + \kappa_P$, needs to be below .12 for calibrated values in the baseline model.

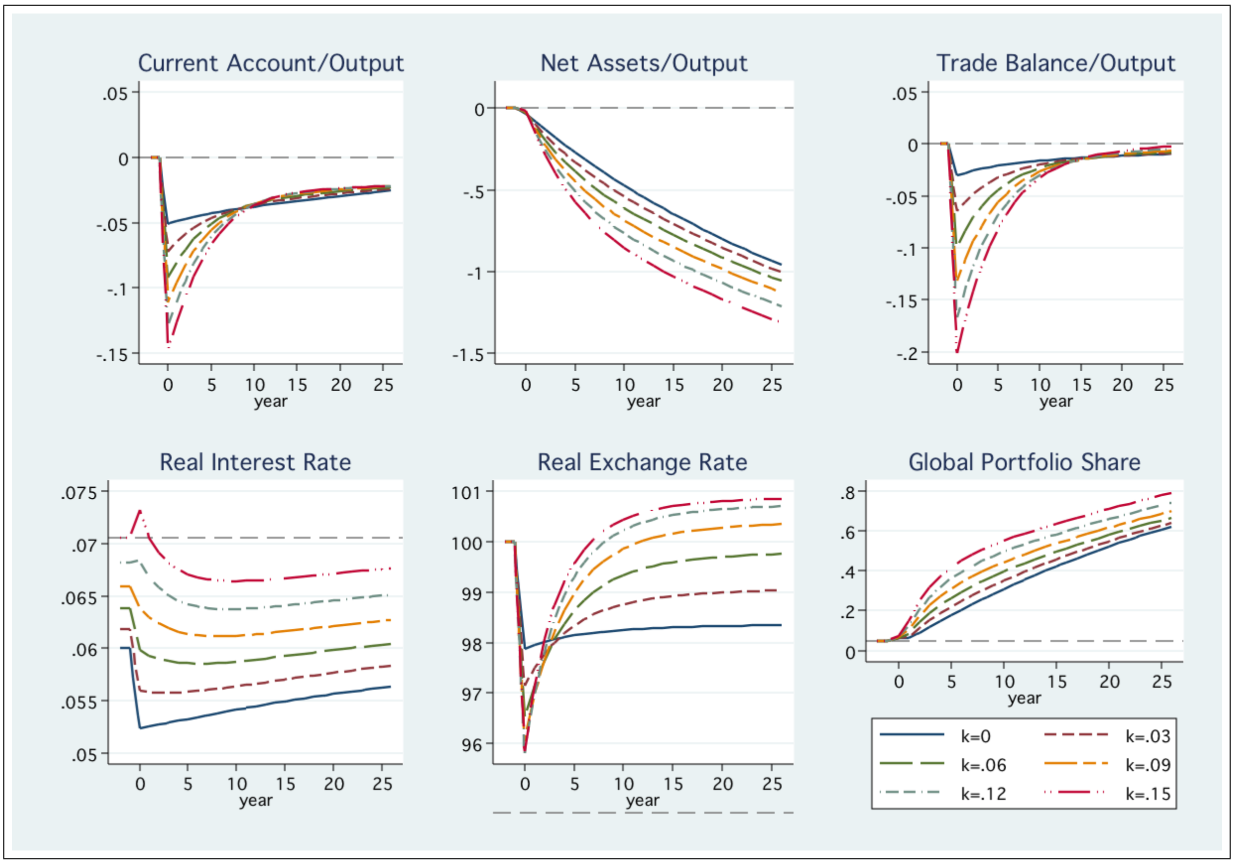


Figure 5: A variation of the net investment parameter κ.

investment costs: The exchange rate needs to depreciate faster and to a lower level in order to sustain the equilibrium path for international debt in the top centre panel of Figure 5. Lower values of κ allow for longer lasting deficits of U's current account.

The model is very sensitive to a variation of κ. Assuming zero costs for investment is clearly a very strong assumption and one that affects the sustainability of debt paths significantly. Since investment costs go hand in hand with the costs for FDI, a joint variation of κ and κ_P is presented below. We will turn to a separate variation of the FDI bargaining price first.

5.3.4. A variation in the FDI bargaining price parameter κ_P

The bargaining price κ_P determines how profits from FDI are shared between the investing country (high δ) and the country the investment takes place in (low δ^R). The smaller the parameter value of κ_P the larger is the share of profits for the investing country. If $\kappa_P = 0$, the investor is able to appropriate the entire gains from lifting the capitalisation rate from δ^R to δ; the price for FDI payable to R becomes $P_t = \kappa_P q_t^R X_t^{Rn} = 0$. Increasing κ_P lifts the amount payable to the country invested in.

The real interest rate panel in Figure 6 brings to light that a variation of κ_P has practically no influence on the real economy as long as investment takes place. This result is straightforward: only the allocation of gains from FDI is affected by κ_P and not the creation of assets itself. If investment costs rise from U's perspective, trade balance and current account deficits are required to finance FDI. These deficits translate directly into larger international indebtedness in net assets over output and a higher global portfolio share. The investing country requires less international debt if foreign investment is less costly and κ_P is lower. At the lower bound with $\kappa_P = 0$, U finances foreign investment entirely from FDI returns and does not require a trade balance deficit to sustain a negative current account balance— the solid blue line in the top right panel of Figure 6.

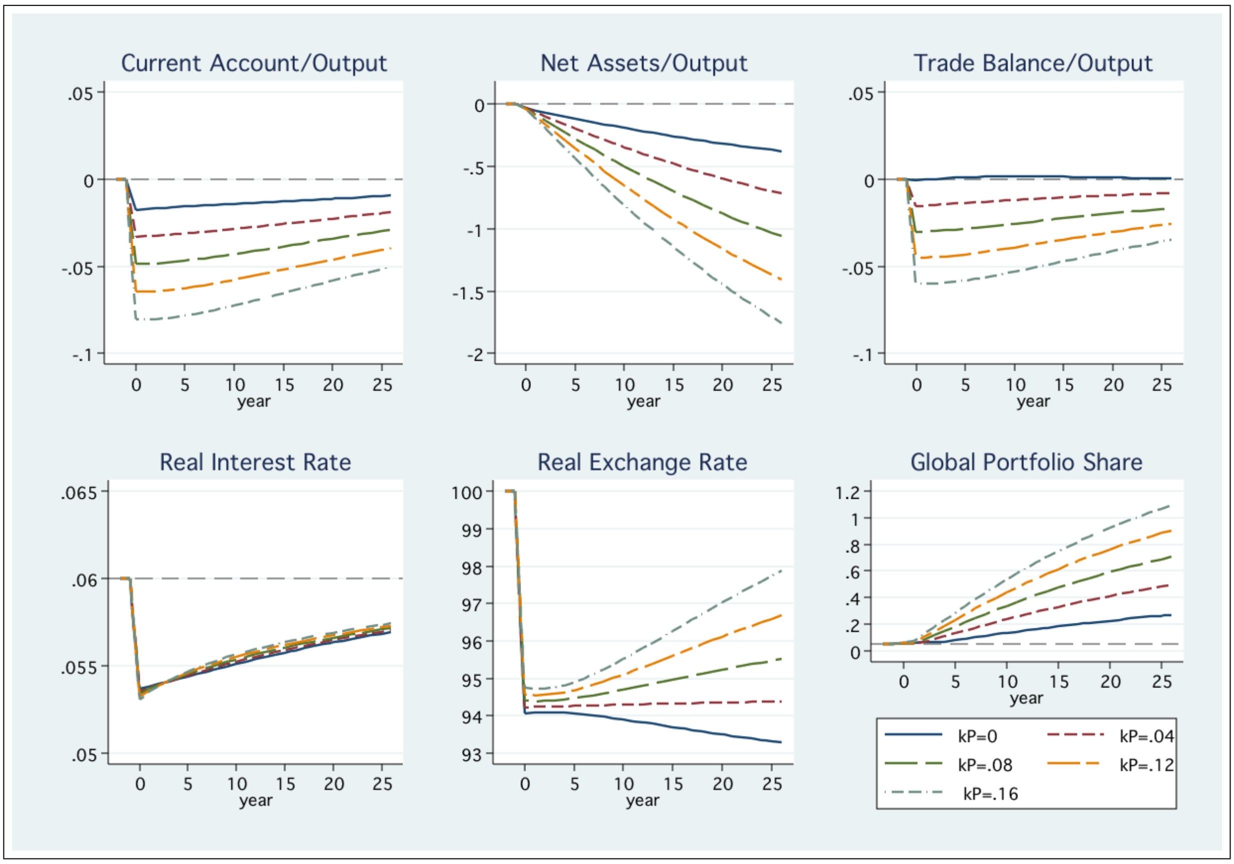


Figure 6: A variation of the FDI bargaining price parameter κ_P.

The exchange rate plays a mediating role: The higher higher κ_P, the more costly is FDI for U. The more money U needs to pay to R, the more depreciated is U's currency through a higher real exchange rate. The FDI bargaining price is therefore a measure of the bargaining power exerted on the investing country. Problematically, the higher the real exchange rate the more unattractive are FDI from the investor's point of view. Depreciation acts like an insulation from too much foreign investment in R.

An extension of this problem is the case when the country invested in (read, some Asian nations) attempts to prevent a real appreciation, but at the same time to lift its lower domestic capitalisation rate δ^R towards the investing country's through FDI. The model tells us that this menu of choice is available if R's real exchange rate can be prevented from appreciating too strongly—the model therefore captures the well-known lamentations about some Asian countries' foreign exchange policies. Next, we will attempt to jointly modify the two investment parameters relating to overall investment costs and its allocation between the partners.

5.3.5. A joint variation of investment costs κ and κ_P

The assumption on asymptotic bilateral private gains from FDI in equation (33) demands the sum $\kappa + \kappa_P$ to lie within the boundaries dictated by the present value of capitalisable assets in each country. With calibrated values as in the baseline case—see Appendix A.1.1 for details—the sum of investment parameters needs to be below 12%. Assuming $\kappa = 0$ and $\kappa_P = .12$ just meets this requirement on the upper bound of greatest benefit from FDI to R. Keeping this upper bound binding, we can vary the parameter values to watch the effect of constant overall investment costs but investment gains accruing differently to the parties involved.

The starting point of investment cost variation in Figure 7 is the baseline model shown by the solid line for $\kappa = 0$ and $\kappa_P = .12$. It now becomes obvious why setting the investment cost parameter κ to zero is so crucial: Any higher value—and correspondingly lower value for κ_P—permits only a considerably lower path of international indebtedness of the investing nation in equilibrium. For the reverse case with high costs of domestic investment and no costs of FDI ($\kappa = .12$, $\kappa_P = 0$), initial trade deficits are high but neither the current account nor the trade balance deficits can be sustained due to the high cost of domestic investment. The investing country is only able to run long-lasting deficits for low values of κ. High investment costs destroy the baseline joint model's favourable outcome and limit persistent current account deficits considerably. That is, in a world of a non-zero net investment rate, high current account deficits simply cannot be upheld persistently.

Foreign investment lies at the heart of the joint model. The combination of an exchange rate and an investment component in a coherent model highlights the versatility of the approach and the lessons which can be drawn from its results. The above derivations allow predictions on current account sustainability in the presence of FDI and exchange rates. These predictions permit policy recommendations which are close to the observed reality of global imbalances. The necessary next step is to re-calibrate the two investment cost parameters and run simulations using more updated values.

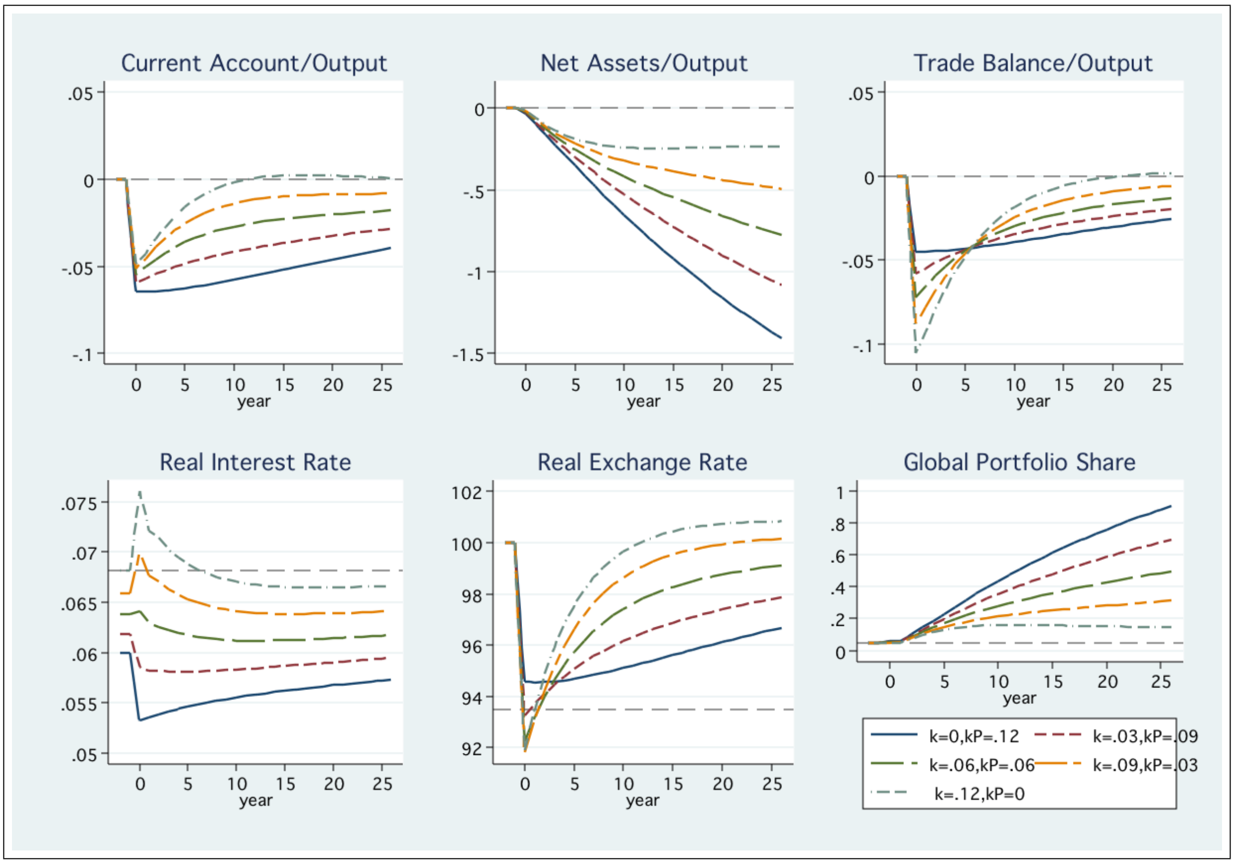


Figure 7: A variation of the incidence of total costs of FDI for $\kappa_P + \kappa = .12$.

5.4. Updated investment costs and reality check

Investment costs are the main driver of the model. The arbitrary assumptions for investment costs made by Caballero et al. (2008) need to be numerically qualified. We can calibrate updated and more realistic values for the cost of domestic and foreign investment using data for the world's largest foreign investor: the US. The value for the domestic net investment rate, κ, is calibrated to 4% (see Appendix A.1.1 for details). Over the past decades, FDI generated a stable rate of return on investment to a US investor of around 4%. The FDI bargaining price is then such that it leaves this return margin of 4% to the investor, translating to $\kappa_P = .12 - .04 = .08$ in the present model; the value of $\kappa_P = .12$ represents the upper bargaining bound from the condition on bilateral private gains in equation (33) which leaves all gains of FDI to the country invested in, i.e. R. Results for the updated calibration are presented for four scenarios.

The baseline scenario from the joint model is given by the solid line in Figure 8. We see the by now well-known characteristics in terms of an initial slump in the interest rate and the reaction of the current account and the trade balance to the shock to δ^R. To make the differences for various values of κ more clearly visible, the real interest rate is normalised to 100. For the low-high scenario, we use the updated calibrated parameter values $\kappa = .04$ (low) and $\kappa_P = .08$ (high). The low-low scenario is for low foreign invest-

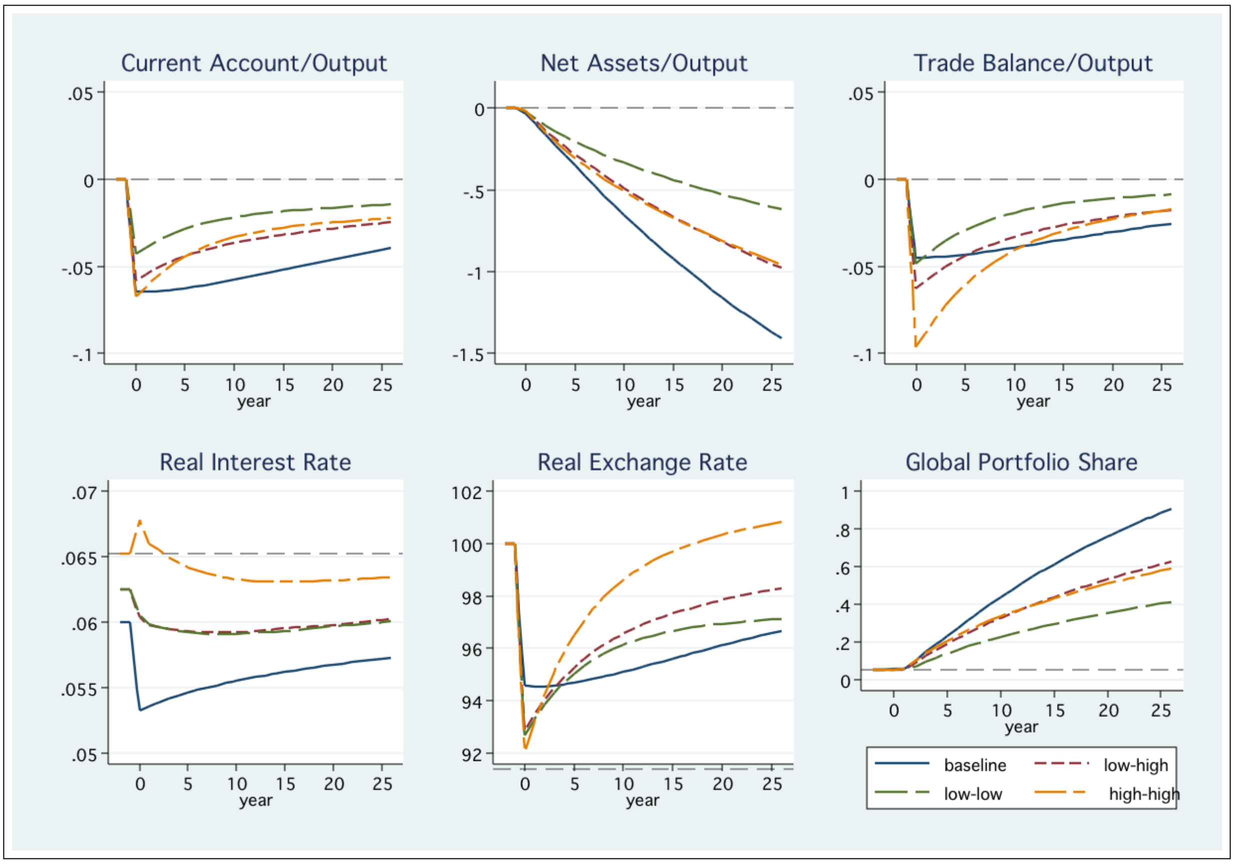


Figure 8: Simulations using realistic assumptions for investment costs κ_P and κ.

ment costs ($\kappa = .04$, $\kappa_P = .04$) while the fourth scenario is the high-domestic investment cost case ($\kappa = .08$, $\kappa_P = .08$). More realistic investment costs force the investing country to sacrifice more resources domestically making the real interest rate decrease less. At the same time, U cannot go as deeply into debt international. This is shown by lower dynamic current account deficits—initial post-shock values in all four scenarious are similar—and a corresponding attenuated net asset output share.

The ultimate reality check is by comparing the first-best calibrated simulation with updated investment cost parameters ($\kappa = .04$, $\kappa_P = .08$) with real-world economic data.[7] Caballero et al. (2008) calibrate the 25% asset market shock to the financial market parameter δ^R at $t = 0$ to the Asian crisis' one in 1998. Figure 9 highlights the relevance of looking at long-term developments: The US development since 1996 resembles the predictions from the updated joint model very closely—possibly closer than may be wished for.

[7] Simulation results are sketched against comparable, annualised time-series from the FRED2 database of the Federal Reserve Bank of St. Louis. The current account/output share uses BOPBCA over GDP, net assets are calculated in the same way as in the simulations using the sum of current account deficits, interest payment on existing net assets and past periods' net assets. The trade balance is BOPGSTB over GDP. The real interest rate is the ten-year treasury rate GS10, because the long-term 30-year rate was unavailable for parts of the sample. The real exchange rate is the trade-weighted exchange index TWEXBMTH minus the annual inflation rate from GDPDEF. All time-series are from 1996 to 2010 with 1998 being $t = 0^+$.

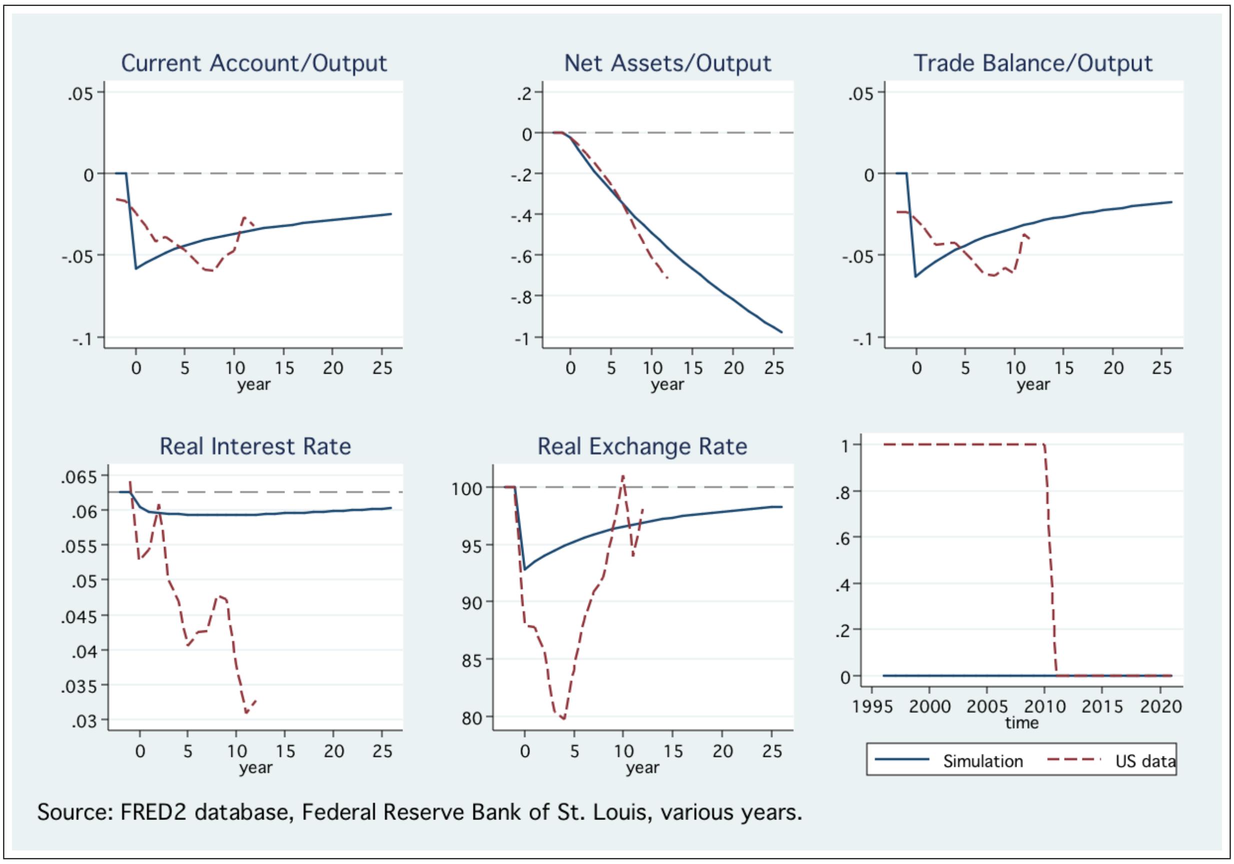


Figure 9: Simulation vs. reality for the joint model with $\kappa = .04$ and $\kappa_P = .08$ and US data (1996–2010).

The current account was in modest deficit at the beginning of the sample period being -1.6% of gross domestic product (GDP) in 1996. Starting with zero net foreign assets in 1996[8], the development of international debt position develops similarly strongly as predicted by calibrated simulations. For 2010, the model predicts a net foreign asset share of –56.3% while in reality it was already at –71.4% of US GDP driven by larger current account deficits than predicted by the model (top left panel). The range of predicted current account and trade balance deficits is similar for both simulation and real data with a small correction following the 2007 crisis. The interest rates for the ten-year treasury bond is comparable to the model interest rate with 6.39% and 6.25% at the start of the sample respectively. However, as a result of the 'Great Moderation', short-term and long-term US interest rates sank to unprecedented lows after the turn of the millennium. Between 1996 and 2002, the trade-weighted index of the US dollar appreciated by 31% while afterwards depreciating 20% until 2010. Adjusted for inflation, real movements are less violent than nominal ones but still much stronger than in the calibrated model as shown in the bottom centre panel of Figure 9. Strikingly—or incidentally—simulation predictions and the real value end up very close together.

[8] The net investment position of the US was zero in 1986 according to BEA estimates (Nguyen 2010, 9) and about abour 5% of GDP in 1996.

In short, the joint model captures the flavour of US international positions after the Asian crisis relatively well given the short time-span for evaluation. All major international indicators—the current account, the net foreign asset position and the trade balance—behave similar to model predictions. Interest rate and exchange rate movements are much stronger than in the model but do not affect the overall outcome shown in the top three panels of Figure 9. In particular, the net asset share is in reality close to the predicted one. The international position of the US is therefore on the extreme frontier of what a theoretical model with perfect foresight would allow. If current account and trade balance deficits are not reduced as in the simulated model, the net foreign asset path becomes unsustainable. The US should behold that they may be on the verge—but they are not yet over it.

6. Concluding remarks

The present paper extends the equilibrium model of global imbalances by Caballero, Farhi, and Gourinchas (2008) by fully incorporating exchange rates and FDI into one coherent model. This joint model captures real-world determinants, of which FDI and exchange rates are an integral part. The model framework allows for three rebalancing channels: (i) the trade balance, (ii) FDI and investment income and (iii) exchange rate adjustment. Calibrated simulations reveal that the joint model is under certain circumstances in line with the benign predictions for sustainable debt levels made for the separate models (Caballero et al. 2008).

The potential of dynamic inconsistencies becomes apparent when parameter values are updated to reflect realistic investment costs. Following a financial market shock, current account balances are driven by two countervailing effects: a slowly declining exchange rate in the deficit country ('slow decline of the dollar') creates positive valuation effects on foreign asset returns. Yet, conducting FDI becomes ever more costly for the investor because its deteriorating terms of trade increase the costs of acquiring foreign assets. Updated calibrations for domestic and foreign investment costs show that sustainable paths of international debt are narrower and shallower for the investing county than predicted by the baseline model.

Sustainability of international debt in the face of shocks to financial markets is not a self-fulfilling prophecy. Rather, the right preconditions need to be met. Given the high sensitivity to parameter changes observed in the simulations, the model is not a blue-print for exculpating deficit nations from making every attempt possible to contain international indebtedness at sustainable levels.

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A. Appendix

A.1. Simulation equations and model dynamics

A.1.1. Updated Calibration

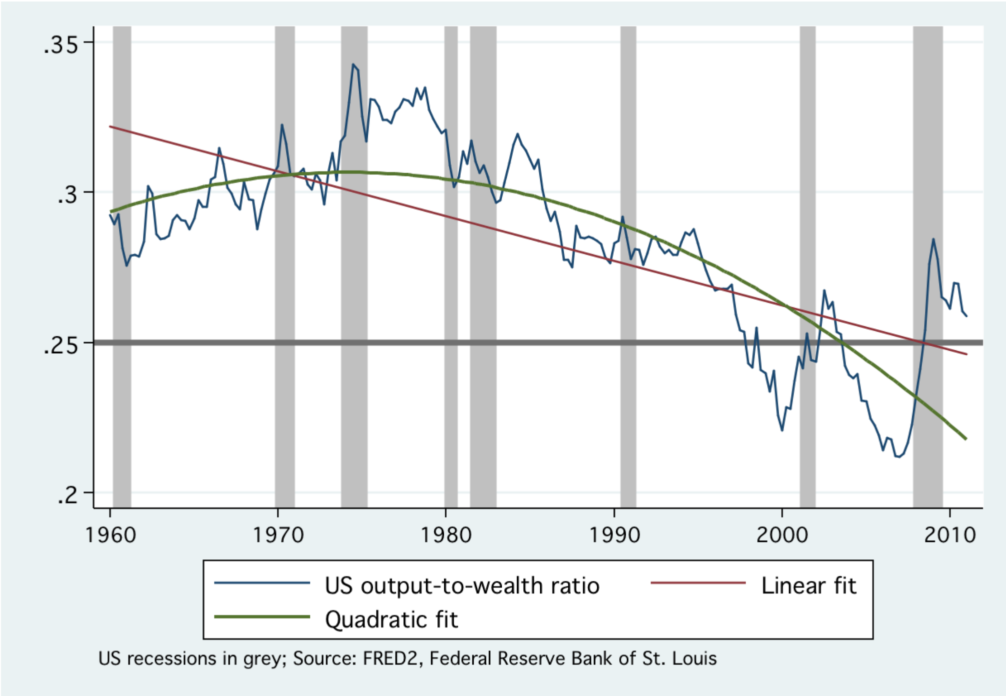
The model simulation uses calibrated starting values analogous to Caballero et al. (2008) which date back to 2004. Some of the value need to be reviewed and updated, however,

Parameter	θ	g	δ	x_0^R	$\mu_{0^-}^{RU}$	$NA_{0^-}^U$	σ	γ	g^z	g^n	κ	κ_P	r_{aut}
Value	0.25	0.03	0.24	0.30	0.05	0.0	4	0.9	0.0	0.03	0.04	0.08	0.06

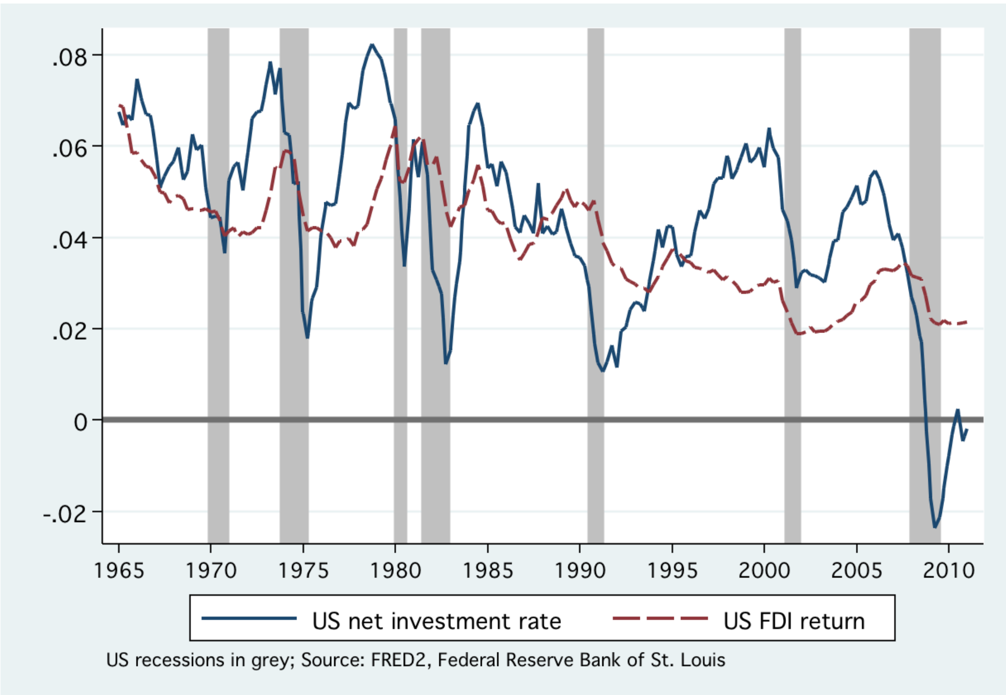
Table A.1: Updated calibrated and starting values for exogenous model parameters.

The parameter θ is approximated by GDP over the net worth of the household sector according to the US Flow of Funds. From Figure 10(a) it is apparent that the dot-com bubble and the housing market bubble both significantly lowered θ compared with the rather stable higher values observed before 1995.

Assuming a value of $\theta = .25$ seems warranted as the mean over the volatile past fifteen or so years. It is depicted in Figure 10(a).



(a) Evolution of the parameter θ (1960-2010).



(b) Evolution of investment (1965-2010).

Figure 10: Calibrating new values for θ, κ and κ_P.

Gross domestic investment is defined by the US National Income and Product Accounts (NIPA) guide as consisting of "'fixed investment and the change in private inventories. Fixed investment consists of both non-residential fixed investment and residential fixed investment. It is measured without a deduction for CSC and includes replacements and additions to the capital stock. [...] It excludes investment by US residents in other countries.'" (BEA 2006, 7) Since the definition of investment in Caballero et al. (2008,

377) reads that "'[p]lanting the $g^n N_t^i$ new trees consumes resources $I_t^i = \kappa q_t^i X_t^i$'", we need net investment, not gross investment. Duca (1997, 5) proposes to deduct NIPA's "'consumption of fixed capital'" in order to get the net investment rate of Figure 10(b). Being extremely volatile in nature, the parameter κ was around 5.2% before 1990, and thereafter exactly 4% until 2007 when it contracted dragging the 20-year average to 3.3%. It seems warranted to assume a long-term net investment rate $\kappa = .04$.

We can get a benchmark for κ_P as the bargaining price for carrying out FDI by looking at the net capital outflows and the resulting stock of US assets abroad. Approximating investment income by "Income Receipts on US Assets Abroad" and "'Other Private Income Receipts on US Assets Abroad'" we can calculate the return on investment for FDI as shown in Figure 10(b). Since 1965 it has slowly decreased to a mean just below 4%. In the model's language, this is the bargaining power of US investors since they derive positive income from abroad. κ_P is therefore 4 percentage points below the value which would render all FDI gains to the foreign country. κ_P is therefore not 12% but $.12 - .04 = .08$.

A.1.2. Nested model properties

All model extensions are based on the same underlying model specification. It is therefore possible to construct a model set-up which incorporates all sub-models by separatating the general equilibrium procedure into four cases: (i) a baseline case without FDI and exchange rates, (ii) the investment case with FDI, (iii) the multiple goods case with exchange rates but no FDI and (iv) a joint model with both FDI and exchange rates. A separation can be achieved by specifying non-adjustable terms of trade $q_t^R = const.$ for $\forall t$. An exchange rate is thus not excluded but the mechanism is disabled as a rebalancing channel and the trade balance and the real interest rate—in (i)—and the interest rate and returns on FDI as in (ii) need to allow for rebalancing. Disabling the FDI component is somewhat more complex since a larger set of equations is affected. In particular, the need to solve for asset values by reverse calculation as shown in appendix A.1.5 becomes unnecessary. The joint model in (iv) is employed as specified in section 4.

Figure 2 in the results section is the graphical exposition of the property that all sub-models are nested within the baseline model. The equations underlying the six panels are given here for comparison:

Current account $CA_t^U = TB_t^U + r_t(\alpha_t^{UR} V_t^R - \alpha_t^{RU} V_t^U) \equiv TB_t^U + r_t NA_t^U$.

Net foreign assets $NA_t^U = \alpha_t^{UR} V_t^R - \alpha_t^{RU} V_t^U \equiv W_t^U - V_t^U$

$$\begin{aligned}
\text{Baseline:}\quad & NA_t^U/X_t^U = \frac{(1-\delta)}{(\theta+g-r_t)} - \frac{\delta}{(r_t-g)}\\
\text{Pure FDI:}\quad & NA_t^U/X_t^U = \frac{(1-\delta-\kappa) + g^n \frac{\delta+\delta x_t^R/x_t^U}{(r_t-g^z)} - (\kappa+\kappa_P)x_t^R/x_t^U}{(\theta+g-r_t)} - \frac{\delta}{(r_t-g)}\\
\text{Pure XR:}\quad & NA_t^U/X_t^U = \frac{(1-\delta)}{(\theta+g-r_t)} - \frac{\delta}{(r_t-g)}\\
\text{Joint model:}\quad & NA_t^U/X_t^U = \frac{(1-\delta-\kappa) + g^n \frac{\delta+\delta x_t^R/x_t^U}{(r_t-g^z)} - (\kappa+\kappa_P)x_t^R/x_t^U}{(\theta+g-r_t)} - \frac{\delta}{(r_t-g)}
\end{aligned}$$

Trade balance $TB_t^U = X_t^U - I_t^U - \theta W_t^U$.

$$
\begin{aligned}
\text{Baseline:} \quad & TB_t^U/X_t^U = 1-\theta\frac{(1-\delta)}{(\theta+g-r_t)} \\
\text{Pure FDI:} \quad & TB_t^U/X_t^U = (1-\kappa)-\theta\frac{(1-\delta-\kappa)+g^n\frac{\delta+\delta x_t^R/x_t^U}{(r_t-g^z)}-(\kappa+\kappa_P)x_t^R/x_t^U}{(\theta+g-r_t)} \\
\text{Pure XR:} \quad & TB_t^U/X_t^U = 1-\theta\frac{(1-\delta)}{(\theta+g-r_t)} \\
\text{Joint model:} \quad & TB_t^U/X_t^U = (1-\kappa)-\theta\frac{(1-\delta-\kappa)+g^n\frac{\delta+\delta x_t^R/x_t^U}{(r_t-g^z)}-(\kappa+\kappa_P)x_t^R/x_t^U}{(\theta+g-r_t)}
\end{aligned}
$$

Real interest rate with $\hat{v}_t^{Ro}q_t^R X_t^{Ro}=V_t^{Ro}$

$$
\begin{aligned}
\text{Baseline:} \quad & r_t = g^z+\theta\left[\delta-(\delta-\delta^R)x_t^R)\right] \\
\text{Pure FDI:} \quad & r_t = g^z+\frac{\theta}{1-\kappa}\left(\left[\delta-(\delta-\delta^R)x_t^R\right]-g^n\hat{v}_t^{Ro}x_t^R\left[\frac{\delta}{\delta^R}-1\right]\right) \\
\text{Pure XR:} \quad & r_t = g^z+\theta\left[\delta-(\delta-\delta^R)x_t^R)\right]+x_t^R\frac{\dot{q}_t^R}{q_t^R} \\
\text{Joint model:} \quad & r_t = g^z+\frac{\theta}{1-\kappa}\left(\left[\delta-(\delta-\delta^R)x_t^R\right]-g^n\hat{v}_t^{Ro}x_t^R\left[\frac{\delta}{\delta^R}-1\right]\right)+x_t^R\frac{\dot{q}_t^R}{q_t^R}
\end{aligned}
$$

Real exchange rate $1=\theta\gamma w_t\left(\gamma+(1-\gamma)q_t^{R(1-\sigma)}\right)^{-1}+(1-\gamma)\left(\frac{(1-\kappa)}{x_t}-\theta w_t\right)\left(\gamma q_t^{R(1-\sigma)}+(1-\gamma)\right)^{-1}$

- Baseline: $q_t^R=const.$, adjustment via r_t and V_{0+}^U.
- Pure FDI: $q_t^R=const.$, adjustment via r_t, FDI and V_{0+}^U.
- Pure XR: $q_t^R=flex.$, adjustment via r_t, q_t^R and V_{0+}^U.
- Joint model: $q_t^R=flex.$, adjustment via r_t, q_t^R, FDI and V_{0+}^U.

Global portfolio share U's assets in R's portfolio holdings as a share of R's overall wealth W_t^R:

$$
\mu_t^{RU}=\alpha_t^{RU}\frac{V_t^U}{W_t^R}=\frac{\sum_{s=0}^{t-1}CA_s^R}{V_t^U}\frac{V_t^U}{W_t^R}=\frac{\sum_{s=0}^{t-1}CA_s^R}{W_t^R}\equiv\frac{\sum_{s=0}^{t-1}\dot{V}_s^U-\dot{W}_s^U}{W_t^R}
$$

A.1.3. Simulation sequence

The source code of the calibrated simulation of the joint model is rather extensive. It is available from the author upon request. A simplified version of the model of a wholly descriptive nature is given in what follows:

Pre-shock ($t=0^-$)

- Starting values and definitions:
  1. Define calibrated parameters as in A.1.4.
  2. Derive pre-shock values for: X_{0-}^U and r_{aut}.
- Iterative solution to q_{0-}^R:
  1. Make initial guess for shock to V_{0+}^R, e.g. a drop by 25%.
  2. Use guess in initial portfolio allocation to derive wealth: $W_{0+}^i=(1-\alpha_{0-}^{ji})V_{0+}^i+\alpha_{0-}^{ij}V_{0+}^j$.
  3. Use shooting algorithm for relative demand equation to solve for (known) X_{0-}^U.
  4. Thereby derive q_{0-}^R, X_{0-}^R and W_{0-}^R using A.1.6.
- Define pre-shock values for all remaining variables requiring terms of trade q_{0-}^R.

Shock $(t = 0^+)$

- Start of main loop from $0^+ \to \infty$ (approximated at $N = 200$ periods/years):
- Solve system $\{w_t, x_t, q_t^R\}$ for $t = 0^+$ using equations in A.1.6:
  - Use post-shock wealth share $w_{0^+} = W_{0^+}^U / X_0^U$ from portfolio allocation estimate.
  - Calculate values for $q_{0^+}^R$ and $x_{0^+}^U$ iteratively until (A.10) converges to 1.
- Solving r_t and $\hat{v}_t^{Ro}$ reversely for $t = \infty \to 0^+$ using the adjusted FDI model in A.1.5.

Post-shock $(0^+ < t \leq \infty)$

- Solve system $\{w_t, x_t, q_t^R\}$ for $t = 1$ using A.1.6:
  - Calculate value for q_1^R and x_1^U iteratively until (A.10) converges to 1.
  - Calculate post-shock rate of change of terms of trade as $\dot{q}_{0^+}/q_{0^+} = (q_1^R - q_{0^+}^R)/q_{0^+}^R$.
  - Initial appreciation at $t = 0^+$ feeds into 'pre-shock' model since $(q_{0^+}^R - q_{0^-}^R)/q_{0^-}^R$ only in $t = 0^-$.

  $\Rightarrow$ No 'over-shooting' of the interest rate at impact because the initial appreciation of $q_{0^+}^R$ does not affect post-shock dynamics.
- Repeat system for $t = 2 \to \infty$ and calculate all other values for remaining variables using one-step iterations.
- End first run-through.

Repeated loop characteristics

- Repeat loop for $(0^+ < t \leq \infty)$ with updated values:
  - Update estimate for $V_{0^+}^R$ using A.1.5.
  - Update estimate for δ^R to produce desired shock: $\delta^R = \delta^R \times (1 - shock) \times V_{0^-}^R / V_{0^+}^R$
- End loop if there is no further change in the guesses for $V_{0^+}^U$ and $V_{0^+}^R$. The system converges.

A.1.4. Initial values before the shock

Initial values for all variables can be created using the equilibrium conditions before the asset market shock to δ^R in $t = 0^+$. The pre-shock period is labelled $t = 0^-$. If a variable is just denoted $t = 0$ then this particular variable is not affected by the shock and remains at the pre-shock value. The whole model is scalable so that absolute values are irrelevant. Only values relative to each region's or total output need to be considered. We can therefore make the following assumptions for the variables describing the real economy $X_t^i = N_t^i Z_t^i$ for all regions $i = \{U, Ro, Rn, R\}$:

$$\begin{aligned} \text{Output:} \quad & X_0^i = x_{0^-}^i \\ \text{Number of trees:} \quad & N_0^i = x_{0^-}^i \\ \text{Productivity:} \quad & Z_0^i = X_0^i / N_0^i = 1 \end{aligned}$$

The calculation of wealth and asset values requires an initial value for the terms of trade $q_{0^-}^R$. The shooting algorithm of the system $\{x_t, w_t, q_t\}$ starts with an initial guess for $q_{0^-}^R$ and thus relative output $x_{0^-} = X_{0^-}^U / (\sum_i q_{0^-}^i X_{0^-}^i)$ and wealth $w_{0^-} = W_{0^-}^U / X_{0^-}^U$ in U. The algorithm is repeated until pasting the previously obtained shooting value for $\tilde{q}^R_{0-}$ delivers the exogenously determined value of X^U_{0-}:

$$\begin{aligned}
\text{Relative demand guess:} \quad & \tilde{X}^U_t = \frac{\theta\gamma\tilde{W}^U_t}{\left(\gamma+(1-\gamma)\tilde{q}^{R(1-\sigma)}_t\right)} + \frac{\theta(1-\gamma)\tilde{W}^R_t}{\left(\gamma\tilde{q}^{R(1-\sigma)}_t+(1-\gamma)\right)} \\
\text{Updated terms of trade guess:} \quad & \tilde{q}^R_t = (X^U_t/\tilde{X}^U_t)q^R_t \\
\text{Updated wealth guess:} \quad & \tilde{W}^R_t = \frac{(1-\delta-\kappa)\tilde{q}^R_t\tilde{X}^U_t}{(\theta-g^z-r_{aut})}
\end{aligned}$$

The shooting value for $\tilde{q}^R_t$ is adjusted so that larger deviations from the target value for X^U_t produce a greater change in the guessed value. The change in the terms of trade are defined as the future period's shooting value minus its current value: $\dot{q}^R_t = q^R_{t+1} - q^R_t$.[9] After obtaining the shooting value for the terms of trade we can then determine initial asset values, wealth and output relations as well as price indices in the respective regions and the real exchange rate between regions i and j:

$$\begin{aligned}
\text{Wealth:} \quad & \frac{W^i_{0-}}{X^i_0} = \frac{1-\delta-\kappa}{\theta-g^z-r_{aut}} \\
\text{Asset values:} \quad & \frac{V^i_{0-}}{X^i_0} = \frac{\delta}{r_{aut}-g^z} \\
\text{Price indices:} \quad & P^i_{0-} = \left(\gamma q^{i(1-\sigma)}_{0-} + (1-\gamma)q^{j(1-\sigma)}_{0-}\right)^{\frac{1}{1-\sigma}} \\
\text{Real exchange rate:} \quad & \lambda^{ij}_{0-} = P^j_{0-}/P^i_{0-}
\end{aligned}$$

In contrast to the pure FDI model, investment in the joint model needs to be adjusted for changes in the terms of trade, too. The bargaining price for executing investment options I^{Rn}_t is susceptible to exchange rate changes according to:

$$\begin{aligned}
\text{Investment:} \quad & I^i_{0-} = \kappa q^i_{0-} X^i_0 \\
\text{Price of FDI:} \quad & P_{0-} = \kappa_P q^i_{0-} X^i_0
\end{aligned}$$

All other variables do not require a particular adjustment to their initial values since their definitions depend only on the above derived parameters. Valuation effects in asset holdings in the α^{ij}_0 and μ^{ij}_{0-} parameters are already accounted for in the asset value and wealth equations.

A.1.5. Derivation of post-shock asset values

In the joint model with FDI and exchange rates, there are three ways of calculating the initial asset values after the shock. All three should ideally give the same result. Due to the iterative nature of the simulation model, small deviations are likely. The derivation of asset values is not as straightforward as in the single model extensions. In the FDI model extension we determined the initial asset value in V^R_{0+} by backward integration using the property that the relative importance of old assets dimishes with time due to non-investment. Using asymptotic asset values for individual assets—$v^{Ro}_t/v^{Rn}_t = \delta^R/\delta$—one could show that $\hat{v}^{Ro}_t = V^{Ro}_t/q^R_t X^{Ro}_t$ would converge to:

$$\hat{v}^{Ro}_\infty = \frac{\delta^R}{\delta}\frac{1}{\theta} \tag{A.1}$$

[9] The exact definition of $\dot{q}^i_t$ is left unspecified by the authors in the final (2008) and all previous working paper versions (2006). None of the original authors replied to requests by the present author to validate the above assumption.

The ratio between old and new asset values, $v_t^{Ro}/v_t^{Rn} = \delta^R/\delta$, is constant over time so that diverging aggregate asset values for V_t^{Ro} and V_t^{Rn} are only driven by investment into the number of assets they are comprised of, $V_t^{Ro} = N_0^R v_t^{Ro}$ and $V_t^{Rn} = (N_t^R - N_0^R)v_t^{Rn}$, respectively. The latter assumes that all investment after the shock to δ^R is undertaken with the aid of U (labelled 'know-how export'). This extreme assumption ensures an upper bound to the effect of FDI in the present model and is maintained for illustrative purposes alone. A partial substitution with effective investment resulting in a mixture of δ^R and δ assets in R is likely in reality but needs additional assumptions for calibrated starting values. The dynamics between this asymptotic value and the time of the shock at $t = 0^+$ are given by:

$$\frac{d\hat{v}_t^{Ro}}{dt} = \frac{\theta}{1-\kappa}\left[\delta(1-x_t^{Ro}) + \delta^R x_t^{Ro} - g^n \hat{v}_t^{Ro} x_t^{Ro}(\delta/\delta^R - 1)\right]\hat{v}_t^{Ro} - \delta^R \tag{A.2}$$

and may be re-written using the interest rate equation derived in (A.11) as:

$$\frac{d\hat{v}_t^{Ro}}{dt} = (r_t - g^z)\hat{v}_t^{Ro} - \delta^R \tag{A.3}$$

The main change is for the reversely solved FDI model to be re-written using exchange rates. In the joint model, the equilibrium interest rate not only depends upon the current value of V_t^{Ro}/X_t^{Ro} which itself depends upon the entire sequence of future interest rates (Caballero et al. 2006, 50). Now, the solution is additionally complicated by incorporating the rate of change of the terms of trade, $\dot{q}_t^R/q_t^R$, for the entire sequence.

Following Caballero et al. (2008, 391), the derivation starts at $t = \infty$ with $x_\infty^{Ro} \approx 0$ and $\hat{v}_t^{Ro} = \hat{v}_\infty^{Ro}$. The value of V_t^{Ro} decreases at the nominal rate g^n due to the non-investment in old assets—$X_t^{Ro} = N_0^R Z_t^R$ for $\forall t$—so that $x_t^{Ro} = q_t^R N_0^R Z_t^R / X_t$ evolves taking logs-and-derivatives of $x_t^{Ro} = q_t^R X_t^{Ro}/\sum q_t^i X_t^i$ as:

$$\begin{aligned}
\dot{x}_t^{Ro}/x_t^{Ro} &= \dot{q}_t^R/q_t^R + g^z - (g x_t^U + g^R x_t^R + x_t^R \dot{q}_t^R/q_t^R) \\
\dot{x}_t^{Ro} &= \left[-g^n + (1-x_t^R)\dot{q}_t^R/q_t^R\right] x_t^{Ro} \\
\frac{\partial x_t^{Ro}}{\partial q_t^R} &= (1-x_t^R)\frac{X_t^{Ro}}{q_t^R X_t^R + X_t^U} = (1-x_t^R)\frac{x_t^{Ro}}{q_t^R}
\end{aligned} \tag{A.4}$$

The system is backwards soluble until $x_t^{Ro} = x_{0^+}^{Ro}$ which is known once we have determined the post-shock real exchange rate. The post-shock output share is then calculated as $x_{0^+}^{Ro} = q_{0^+}^R X_0^{Ro}/X_{0^+}$. It is therefore necessary to employ the shooting algorithm in A.1.6 to solve for the real exchange rate and use the derived value for $q_{0^+}^R$ in the above model until it converges. Convergence of the model is assumed once a changes in the real exchange rate do not affect post-shock asset values and *vice versa*.

The necessary condition for convergence is for the FDI-driven process to produce the same post-shock asset value $V_{0^+}^R$ as the one using the present value derivation from the exchange rate model. For this, we need two conditions to be met: $V_{0^+}^R$ needs to be equal in both cases and δ^R needs to be calibrated so as to produce the desired shock at $t = 0^+$, in our case $\Delta V_{0^+}^R / V_{0^-}^R = -25\%$.

By definition: $$V_{0^+}^R = N_0^R v_{0^+}^{Ro} = N_0^R \delta^R q_{0^+}^R Z_{0^+}^R \int_{0^+}^{\infty} e^{-\int_{0^+}^{s}(r_u - g^z)du} ds \tag{A.5}$$

From the XR model: $$\begin{aligned} V_{0^+}^R &= V_{0^+}^{Ro} + V_{0^+}^{Rn} \\ &= \delta^R \int_0^\infty q_t^R X_t^{Ro} e^{-\int_0^s r_u du} ds + \delta \int_0^\infty q_t^R X_t^{Rn} e^{-\int_0^s r_u du} ds \end{aligned} \tag{A.6}$$

From the FDI model: $$V_{0^+}^R = \hat{v}_{0^+}^{Ro} q_{0^+}^R X_{0^+}^{Ro} \tag{A.7}$$

All three approaches to evaluate $V_{0^+}^{Ro}$ should therefore yield the same result. The financial market parameter δ^R is the factor which brings this equality about since it is iteratively adjusted until $V_{0^+}^{Ro} =$

$(1-shock)\times V^{Ro}_{0^-}$.

A.1.6. Post-shock exchange rate dynamics

The shock in $t=0^+$ needs to be calibrated so as to produce a decline in asset values of 25% in response to a reduction in δ^R. The simulations of the FDI and exchange rate extensions each separately posed the additional difficulty of having to solve for initial asset values $V^i_{0^+}$ using backward and forward integration respectively. In the joint model, we still know that in $x^{Ro}_\infty = 0$ and additionally $\dot{q}^R_\infty/q^R_\infty = 0$ but we cannot postulate a sequence for $\dot{x}^{Ro}_t$ as in the separate model due to incomplete knowledge of the future path of the terms of trade affecting relative output. Similarly, we cannot find the post-shock value of $V^R_{0^+}$ for lack of knowledge of the evolution of relative output and the rate of interest. We therefore have to solve the model—which is nonetheless uniquely identified and only lacks starting values—by using a shooting algorithm.

The model is iteratively solved back and forth until a coherent, i.e. non divergent, path for r_t, q^R_t and V^i_t is found. These paths include the solution to the system (w_t, x_t, q^i_t) from the exchange rate extension. The system includes the shorthands for $w_t = W^U_t/X^U_t$, $x_t = X^U_t/\sum q^i_t X^i_t$ and $\gamma = \gamma_{UU} = 1-\gamma_{UR}$ from U's perspective. The major change manifests itself in wealth dynamics and in the interest rate equations which is additionally dependent upon the evolution of old and new asset values in R:

$$\text{Wealth dynamics:} \quad \dot{w}_t = (r_t-\theta-g)w_t + (1-\delta-\frac{\kappa}{x_t}) + g^n\left[\frac{(1-\kappa)}{\theta x_t} + \hat{v}^{Ro}_t\frac{x^{Ro}_t}{x_t}\left(\frac{\delta}{\delta^R}-1\right)\right] - \kappa_p\frac{x^{Rn}_t}{x_t} \tag{A.8}$$

$$\text{Terms of trade:} \quad 1 = \theta\gamma w_t P^{U(\sigma-1)}_t + (1-\gamma)\left(\frac{(1-\kappa)}{x_t} - \theta w_t\right)P^{R(\sigma-1)}_t \tag{A.9}$$

$$\text{Output dynamics:} \quad \dot{x}_t = x_t(1-x_t)\left(g-g^R-\frac{\dot{q}^R_t}{q^R_t}\right) \tag{A.10}$$

$$\text{Interest rate:} \quad r_t = g^z + x^R_t\frac{\dot{q}^R_t}{q^R_t} + \frac{\theta}{(1-\kappa)}\left[\delta-(\delta-\delta^R)x^{Ro}_t - g^n\hat{v}^{Ro}_t x^R_t\left(\frac{\delta}{\delta^R}-1\right)\right] \tag{A.11}$$

The wealth dynamics equation (A.8) is a variation of the global asset demand equation (37). It can be derived as follows:

$$\begin{aligned}
\dot{W}^U_t &= (r_t-\theta)W^U_t + (1-\delta)X^U_t + g^n(V^U_t + N^R_t v^{Rn}_t) - \kappa_p q^R_t X^{Rn}_t - \kappa X_t \\
\dot{w}_t &= (r_t-\theta-g)w_t + (1-\delta) + g^n(V^U_t + N^R_t v^{Rn}_t)/\dot{X}^U_t - \kappa_p x^{Rn}_t/x_t - \kappa/x_t \\
&= (r_t-\theta-g)w_t + (1-\delta) + g^n(V_t - N^R_0(v^{Ro}_t - v^{Rn}_t))/\dot{X}^U_t - \kappa_p x^{Rn}_t/x_t - \kappa/x_t \\
&= (r_t-\theta-g)w_t + (1-\delta) + g^n(V_t + N^R_0 v^{Ro}_t(\delta/\delta^R-1))/\dot{X}^U_t - \kappa_p x^{Rn}_t/x_t - \kappa/x_t \\
&= (r_t-\theta-g)w_t + (1-\delta) + g^n(V_t + V^{Ro}_t(\delta/\delta^R-1))/\dot{X}^U_t - \kappa_p x^{Rn}_t/x_t - \kappa/x_t \\
\dot{w}_t &= (r_t-\theta-g)w_t + (1-\delta) + g^n((1-\kappa)/(\theta x_t) + \hat{v}^{Ro}_t x^{Ro}_t/x_t(\delta/\delta^R-1)) - \kappa_p x^{Rn}_t/x_t - \kappa/x_t
\end{aligned}$$

The terms of trade equation (A.9) utilises the equilibrium condition on the goods market using $P^iC^i =$

θW^i:

$$\begin{aligned}
X_t^U &= \sum_i \gamma_{ij} C^i \left(\frac{q^j}{Pi}\right)^{-\sigma} \\
&= \gamma\theta W_t^U P_t^{U(\sigma-1)} + (1-\gamma)\theta W_t^R P_t^{R(\sigma-1)} \\
&= \gamma\theta W_t^U P_t^{U(\sigma-1)} + (1-\gamma)\theta (W_t - W_t^U) P_t^{R(\sigma-1)} \\
&= \gamma\theta W_t^U P_t^{U(\sigma-1)} + (1-\gamma)\theta \left(\frac{(1-\kappa)}{\theta} X_t - W_t^U\right) P_t^{R(\sigma-1)} \\
\frac{X_t^U}{X_t^U} &= \theta\gamma w_t P_t^{U(\sigma-1)} + (1-\gamma)\left(\frac{(1-\kappa)}{x_t} - \theta w_t\right) P_t^{R(\sigma-1)} \\
1 &= \theta\gamma w_t \left(\gamma + (1-\gamma) q_t^{R(1-\sigma)}\right)^{-1} + (1-\gamma)\left(\frac{(1-\kappa)}{x_t} - \theta w_t\right)\left(\gamma q_t^{R(1-\sigma)} + (1-\gamma)\right)^{-1}
\end{aligned}$$

with $P_t^i = \left(\sum_j \gamma_{ij} q_t^{j(1-\sigma)}\right)^{1/(1-\sigma)}$ for $j = \{U, R\}$. A shooting mechanism is used to find the value for q_t^R for values of w_t and x_t derived using the dynamic equations of the preceding period. Final values are obtained using the above equations while the guess for V_{0+}^i is iteratively updated every time the shooting algorith reaches the end of the loop at $t = \infty$:

$$\begin{aligned}
\text{Guess for asset values: } V_{0+}^i &= \delta^i \int_0^\infty q_t^i X_t^i e^{-\int_0^s r_u du} ds \\
&= q_0^i X_0^i \delta^i \int_0^\infty e^{-\theta \int_0^s \bar{\delta}_u du} \frac{x_s^i}{x_0^i} ds
\end{aligned}$$

The average capitalisation ratio in this equation is time-varying since relative output is constantly changing, $\bar{\delta} = \sum_i x_t^i \delta^i$, and it is also dependent upon exchange rate changes. For the combined case, we need a new estimation for V_{0+}^R since it now consists of the old and new trees exhibiting different capitalisation rates δ^i. The initial guess for their aggregate asset value is:

$$\begin{aligned}
V_{0+}^R &= V_{0+}^{Ro} + V_{0+}^{Rn} \\
&= \delta^R \int_0^\infty q_t^R X_t^{Ro} e^{-\int_0^s r_u du} ds + \delta \int_0^\infty q_t^R X_t^{Rn} e^{-\int_0^s r_u du} ds \qquad \text{(A.12)}
\end{aligned}$$

The values for V_{0+}^{Ro} and V_{0+}^{Rn} have to correspond to the values obtained in the reverse solution using the asymptotic property $\lim_{t\to\infty} x^{Ro} = 0$. The derivation is described in section A.1.5.

A.2. Further model results

Table A.2: Results summary for variations of baseline.

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
joint model, FDI, XR	-2	0	0	0	.06	1.133215	.05
	0	-.064441	-.032315	-.044972	.053239	1.071662	.057158
	3	-.063865	-.225319	-.044399	.054138	1.071777	.144091
	10	-.057735	-.651972	-.03931	.05552	1.077677	.436608
	15	-.051964	-.922049	-.034775	.056197	1.083451	.613499
	50	-.020443	-2.04959	-.012697	.058673	1.113919	1.258188
pure FDI, noXR	-2	0	0	0	.06	1.133215	.05

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... table A.2 continued

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
	0	-.048704	-.036479	-.012857	.051038	1.133215	.058779
	3	-.062838	-.19958	-.030625	.051824	1.133215	.121789
	10	-.065658	-.656524	-.036562	.053393	1.133215	.410001
	15	-.060342	-.96666	-.032785	.054321	1.133215	.600987
	50	-.024206	-2.265196	-.01076	.05801	1.133215	1.352625
pure XR, noFDI	-2	0	0	0	.06	1.133215	.05
	0	-.048476	-.027935	-.0406	.055767	1.087723	.054674
	3	-.025194	-.146422	-.010632	.055577	1.127835	.086095
	10	-.004939	-.245635	.015804	.055425	1.162521	.13635
	15	-.001185	-.261467	.020954	.055397	1.1691	.141637
	50	.000151	-.257653	.02373	.055381	1.172629	.13661
baseline, noFDI, noXR	-2	0	0	0	.06	1.133215	.05
	0	-.023694	-.030596	0	.055381	1.133215	.055348
	3	-.01487	-.091593	.011334	.055381	1.133215	.053463
	10	-.00734	-.166818	.021007	.055381	1.133215	.091263
	15	-.005938	-.200119	.022809	.055381	1.133215	.108866
	50	-.005219	-.38648	.023731	.055381	1.133215	.209608

Table A.3: Results summary for variations of deltaR.

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
deltaR dR=9	-2	0	0	0	.06	1.133215	.05
	0	-.129492	-.071014	-.104168	.049707	1.016128	.062406
	3	-.107271	-.43568	-.077378	.048232	1.051595	.279495
	10	-.082106	-1.104198	-.044609	.0485	1.091709	.708437
	15	-.073239	-1.497039	-.033011	.049596	1.103707	.941837
	50	-.034314	-3.140224	-.007305	.056209	1.124757	1.872805
deltaR dR=12	-2	0	0	0	.06	1.133215	.05
	0	-.10729	-.05797	-.084057	.050903	1.034797	.060504
	3	-.092495	-.363997	-.066024	.050215	1.058587	.232999
	10	-.073892	-.949004	-.042463	.050906	1.087263	.616955
	15	-.066052	-1.299075	-.033287	.051851	1.097196	.83181
	50	-.029443	-2.769258	-.009121	.057035	1.121213	1.667109
deltaR dR=15	-2	0	0	0	.06	1.133215	.05
	0	-.085769	-.045152	-.064427	.052066	1.053268	.058787
	3	-.078132	-.294374	-.055071	.052175	1.065307	.188247
	10	-.065816	-.799371	-.040691	.053242	1.082625	.527128
	15	-.059021	-1.108892	-.033887	.054038	1.090449	.723349

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... table A.3 continued

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
	50	-.024849	-2.40803	-.010908	.057854	1.117616	1.463619
deltaR dR=18	-2	0	0	0	.06	1.133215	.05
	0	-.064712	-.032474	-.045199	.053228	1.07144	.057182
	3	-.064038	-.226188	-.044512	.054112	1.071729	.144648
	10	-.057809	-.653814	-.039289	.055506	1.077787	.437772
	15	-.052077	-.924473	-.034784	.056162	1.083505	.61496
	50	-.020494	-2.054063	-.012675	.058663	1.113964	1.260785
deltaR dR=21	-2	0	0	0	.06	1.133215	.05
	0	-.043914	-.019872	-.026308	.054409	1.089206	.055626
	3	-.05009	-.15896	-.034351	.056052	1.077799	.101746
	10	-.049858	-.510988	-.038303	.05768	1.072661	.34773
	15	-.045132	-.743878	-.035933	.058252	1.076415	.505095
	50	-.01635	-1.704834	-.014433	.059466	1.110265	1.056981
deltaR dR=24	-2	0	0	0	.06	1.133215	.05
	0	.00155	.000333	.002039	.06	1.135894	.049956
	3	.000824	.003993	.001084	.06	1.134656	.047682
	10	.000188	.007205	.000248	.06	1.133551	.045766
	15	.000066	.007826	.000086	.06	1.133337	.045415
	50	0	.008158	0	.06	1.133215	.045241
deltaR dR=27	-2	0	0	0	.06	1.133215	.05
	0	.02543	.014237	.021547	.062313	1.156531	.047667
	3	.013652	.074258	.005883	.062266	1.136602	.011415
	10	.0033	.127743	-.007773	.062226	1.11852	0
	15	.001263	.138699	-.010397	.062218	1.115015	0
	50	.000027	.147386	-.011791	.062214	1.113144	0

Table A.4: Results summary for variations of sigma.

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
sigma s=37	-2	0	0	0	.06	100	.05
	0	-.063318	-.032254	-.043779	.053273	94.201508	.056738
	3	-.063043	-.222198	-.043584	.054223	94.160782	.142013
	10	-.057254	-.64415	-.039027	.055673	94.654938	.433578
	15	-.051513	-.911862	-.03464	.056319	95.205086	.610328
	50	-.020232	-2.030273	-.012837	.058725	98.118965	1.249827
sigma s=40	-2	0	0	0	.06	100	.05
	0	-.064441	-.032315	-.044972	.053239	94.568291	.057158
	3	-.063865	-.225318	-.044399	.054138	94.578453	.144091

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... table A.4 continued

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
	10	-.057735	-.651969	-.039309	.055522	95.09893	.436605
	15	-.051964	-.922053	-.034775	.056196	95.608704	.613503
	50	-.020443	-2.04959	-.012697	.058673	98.297264	1.258188
sigma s=50	-2	0	0	0	.06	100	.05
	0	-.067674	-.032553	-.048238	.053107	95.500305	.05834
	3	-.066174	-.234258	-.046511	.053879	95.620964	.150083
	10	-.059071	-.673805	-.039933	.055187	96.161964	.445657
	15	-.053054	-.950204	-.034904	.055887	96.597954	.623388
	50	-.020993	-2.101665	-.012346	.058549	98.710075	1.281788
sigma s=80	-2	0	0	0	.06	100	.05
	0	-.074226	-.033254	-.054287	.052726	97.003639	.060672
	3	-.070659	-.252312	-.049963	.053354	97.220703	.162338
	10	-.061602	-.716189	-.04063	.054618	97.691483	.465232
	15	-.055126	-1.004156	-.034865	.055365	97.987427	.645692
	50	-.021947	-2.196604	-.011801	.058359	99.258179	1.327997
sigma s=120	-2	0	0	0	.06	100	.05
	0	-.079194	-.033936	-.058483	.05236	97.916473	.062426
	3	-.073957	-.265984	-.052058	.052959	98.131271	.171736
	10	-.063465	-.747372	-.040862	.054254	98.500412	.480843
	15	-.056659	-1.043482	-.034701	.055045	98.704025	.663922
	50	-.022577	-2.262859	-.011479	.058249	99.528885	1.362032

Table A.5: Results summary for variations of kappa.

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
kappa k=0	-2	0	0	0	.06	100	.05
	0	-.050839	-.03388	-.030509	.05239	97.87207	.062537
	3	-.045415	-.180204	-.023891	.052893	98.065834	.116685
	10	-.037808	-.470803	-.016216	.054137	98.250427	.307478
	15	-.033673	-.647027	-.01336	.054936	98.300346	.422379
	50	-.011752	-1.347712	-.006015	.058216	98.403954	.874183
kappa k=3	-2	0	0	0	.061856	100	.05
	0	-.072201	-.029508	-.06499	.055983	97.14431	.063115
	3	-.053968	-.22553	-.041802	.055751	97.99955	.151382
	10	-.036474	-.538846	-.019773	.05637	98.758247	.351454
	15	-.031297	-.707933	-.01416	.057033	98.92141	.457825
	50	-.012226	-1.390942	-.004798	.060136	99.118156	.887793
kappa k=6	-2	0	0	0	.06383	100	.05

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... table A.5 continued

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
	0	-.092149	-.02552	-.098813	.059837	96.5121	.064086
	3	-.062674	-.268926	-.059979	.058788	98.04528	.187434
	10	-.035727	-.609225	-.023373	.058739	99.336723	.396443
	15	-.029439	-.774668	-.014803	.059263	99.595505	.495645
	50	-.012903	-1.452096	-.003261	.06218	99.860458	.9107
kappa k=9	-2	0	0	0	.065934	100	.05
	0	-.111062	-.021886	-.132559	.063977	96.047775	.065693
	3	-.071751	-.311616	-.078672	.06192	98.204872	.226108
	10	-.035554	-.683985	-.026821	.061208	99.855782	.443922
	15	-.028105	-.849336	-.015105	.061615	100.16175	.537501
	50	-.0138	-1.534117	-.001339	.064357	100.44504	.945703
kappa k=12	-2	0	0	0	.068182	100	.05
	0	-.129143	-.018574	-.166664	.068439	95.807251	.068249
	3	-.081367	-.354385	-.098018	.065067	98.448967	.268661
	10	-.035946	-.764616	-.029982	.063768	100.22785	.495103
	15	-.027301	-.933511	-.014968	.064091	100.5313	.584638
	50	-.014914	-1.638808	.001015	.066677	100.79386	.99409
kappa k=15	-2	0	0	0	.070588	100	.05
	0	-.146341	-.015538	-.201368	.07328	95.837883	.072184
	3	-.09159	-.397623	-.118034	.068177	98.738197	.316351
	10	-.0369	-.851918	-.032826	.066433	100.43659	.550677
	15	-.027026	-1.028103	-.014376	.066699	100.70654	.637595
	50	-.016238	-1.767018	.003845	.069151	100.92847	1.055737

Table A.6: Results summary for variations of kappaP.

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
kappaP kP=0	-2	0	0	0	.06	100	.05
	0	-.017698	-.031492	-.000975	.053696	94.066574	.056759
	3	-.016367	-.08228	.000482	.0541	94.087044	.061613
	10	-.014247	-.189411	.001505	.055106	93.897797	.131058
	15	-.012827	-.256466	.001332	.055757	93.699539	.180546
	50	-.002508	-.501203	-.002487	.058509	92.674026	.367497
kappaP kP=4	-2	0	0	0	.06	100	.05
	0	-.033202	-.031754	-.015597	.053547	94.228394	.056886
	3	-.032126	-.130017	-.01444	.054113	94.246246	.083497
	10	-.028584	-.343219	-.011971	.055237	94.292847	.234907
	15	-.025641	-.477356	-.010499	.055902	94.324158	.329756

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... table A.6 continued

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
	50	-.008161	-1.006369	-.005666	.058555	94.454086	.695094
kappaP kP=8	-2	0	0	0	.06	100	.05
	0	-.048776	-.032026	-.03026	.053397	94.396393	.057017
	3	-.047955	-.177547	-.0294	.054126	94.410187	.113782
	10	-.043081	-.497245	-.025582	.055366	94.694374	.336637
	15	-.038685	-.699006	-.022535	.056056	94.959763	.473869
	50	-.014129	-1.522127	-.009064	.058611	96.330856	.99087
kappaP kP=12	-2	0	0	0	.06	100	.05
	0	-.064441	-.032315	-.044972	.053239	94.568283	.057158
	3	-.063865	-.225318	-.044399	.054138	94.578438	.144091
	10	-.057735	-.651969	-.039309	.055522	95.09893	.436605
	15	-.051964	-.922053	-.034775	.056196	95.608734	.613502
	50	-.020443	-2.049589	-.012697	.058673	98.297249	1.258187
kappaP kP=16	-2	0	0	0	.06	100	.05
	0	-.080179	-.032611	-.059739	.053076	94.745148	.057302
	3	-.07985	-.273153	-.059449	.054149	94.749481	.174305
	10	-.072558	-.807186	-.053187	.05566	95.508873	.534702
	15	-.065486	-1.146059	-.047234	.056356	96.264702	.748477
	50	-.027129	-2.588811	-.016574	.058742	100.34383	1.499469

Table A.7: Results summary for variations of kappa kappaP.

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
kappa kappaP k=0,kP=12	-2	0	0	0	.06	100	.05
	0	-.064441	-.032315	-.044972	.053239	94.568283	.057158
	3	-.063865	-.225319	-.044399	.054138	94.578491	.144091
	10	-.057735	-.651972	-.03931	.05552	95.099136	.436608
	15	-.051964	-.922049	-.034775	.056197	95.60862	.613499
	50	-.020443	-2.04959	-.012697	.058673	98.297272	1.258188
kappa kappaP k=3,kP=9	-2	0	0	0	.061856	100	.05
	0	-.059654	-.025473	-.058366	.058554	93.225632	.053984
	3	-.05229	-.196692	-.048763	.058116	94.455681	.128759
	10	-.041711	-.527981	-.03476	.058295	96.151932	.35386
	15	-.036756	-.725894	-.028595	.058672	96.865707	.480746
	50	-.015504	-1.5759	-.009275	.060664	99.076637	.975945
kappa kappaP k=6,kP=6	-2	0	0	0	.06383	100	.05
	0	-.055095	-.019794	-.072361	.064108	92.259163	.051568
	3	-.042214	-.171649	-.053852	.062223	94.65316	.11525

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... table A.7 continued

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
	10	-.027204	-.417274	-.030028	.061193	97.410896	.279252
	15	-.022825	-.547372	-.021944	.061218	98.261711	.359905
	50	-.010897	-1.119327	-.005576	.062773	99.826202	.699257
kappa kappaP k=9,kP=3	-2	0	0	0	.065934	100	.05
	0	-.05135	-.015051	-.08788	.069924	91.784058	.049981
	3	-.033807	-.151468	-.059949	.066328	95.163132	.104666
	10	-.013963	-.320837	-.024767	.064138	98.639671	.21412
	15	-.009859	-.386503	-.014412	.063844	99.527214	.252119
	50	-.006616	-.682891	-.001634	.065013	100.4047	.430936
kappa kappaP k=12,kP=0	-2	0	0	0	.068182	100	.05
	0	-.048753	-.011068	-.105657	.076035	91.862312	.04932
	3	-.027235	-.137189	-.067167	.070272	95.904083	.098048
	10	-.001798	-.239805	-.01866	.067036	99.641647	.159618
	15	.002147	-.244202	-.005942	.066532	100.44228	.158157
	50	-.002659	-.272466	.002537	.067389	100.75764	.174035

Table A.8: Results summary for variations of growth.

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
growth g=0	-2	0	0	0	.06	1.133215	.05
	0	-.075491	-.020321	-.08866	.060786	1.076423	.050173
	3	-.063432	-.234348	-.071119	.058993	1.106187	.137618
	10	-.049766	-.635528	-.047968	.057117	1.143916	.357842
	15	-.046876	-.886991	-.040357	.056486	1.156015	.488647
	50	-.051355	-2.658295	-.032117	.055834	1.168838	1.422098
growth g=1	-2	0	0	0	.06	1.133215	.05
	0	-.071306	-.023949	-.073464	.058286	1.07601	.052271
	3	-.062701	-.228871	-.061623	.057233	1.095231	.138196
	10	-.052473	-.635139	-.045706	.056381	1.120533	.37927
	15	-.049272	-.894892	-.039716	.056284	1.130031	.525921
	50	-.039333	-2.454903	-.023575	.057103	1.156187	1.355366
growth g=2	-2	0	0	0	.06	1.133215	.05
	0	-.067773	-.027775	-.059623	.055863	1.073208	.054514
	3	-.062856	-.225977	-.053279	.055689	1.082877	.140505
	10	-.055027	-.640579	-.043159	.055916	1.098014	.405672
	15	-.050869	-.906303	-.038126	.056222	1.105489	.567674
	50	-.029102	-2.249187	-.017522	.058045	1.13639	1.304029
growth g=3	-2	0	0	0	.06	1.133215	.05

Continued on next page...

... table A.8 continued

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
	0	-.064517	-.031811	-.046586	.053518	1.069059	.0569
	3	-.063469	-.224523	-.045439	.054333	1.070052	.143843
	10	-.056999	-.64736	-.039742	.055634	1.076907	.434291
	15	-.051244	-.914292	-.03505	.056283	1.082952	.609158
	50	-.020213	-2.030648	-.012772	.058699	1.113797	1.246972
growth g=4	-2	0	0	0	.06	1.133215	.05
	0	-.061352	-.036089	-.034024	.051237	1.064131	.05944
	3	-.064311	-.223933	-.037685	.053144	1.057279	.14785
	10	-.05816	-.653169	-.03511	.055472	1.057509	.463417
	15	-.050362	-.915832	-.03036	.056434	1.062473	.647445
	50	-.012316	-1.798828	-.008642	.059141	1.090925	1.174038

Table A.9: Results summary for variations of sizeN.

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
sizeN N=100	-2	0	0	0	.06	100	.05
	0	-.064441	-.032315	-.044972	.053239	94.568283	.057158
	3	-.063865	-.225319	-.044399	.054138	94.578491	.144091
	10	-.057735	-.651972	-.03931	.05552	95.099136	.436608
	15	-.051964	-.922049	-.034775	.056197	95.60862	.613499
	50	-.020443	-2.04959	-.012697	.058673	98.297272	1.258188
sizeN N=150	-2	0	0	0	.06	100	.05
	0	-.064511	-.031834	-.046509	.053504	94.349442	.056911
	3	-.063488	-.224555	-.045392	.054322	94.433189	.143851
	10	-.057036	-.647586	-.039728	.055616	95.035172	.434407
	15	-.051284	-.914619	-.035042	.056287	95.564354	.609325
	50	-.020223	-2.031523	-.012768	.058698	98.287033	1.24749
sizeN N=200	-2	0	0	0	.06	100	.05
	0	-.064513	-.03181	-.046586	.053518	94.338745	.056899
	3	-.063468	-.224513	-.045442	.054331	94.425987	.143836
	10	-.057	-.647355	-.039748	.055622	95.031975	.434289
	15	-.051249	-.914234	-.035056	.056291	95.562302	.609107
	50	-.020212	-2.0306	-.012772	.058699	98.286522	1.246943

Table A.10: Results summary for variations of new kappas.

Variable Names	period	CAU XU	NAU XU	TBU XU	r t	lambdaUR	muRU
new kappas k=0,kP=12	-2	0	0	0	.06	100	.05
	0	-.064441	-.032315	-.044972	.053239	94.568283	.057158
	3	-.063865	-.225319	-.044399	.054138	94.578491	.144091
	10	-.057735	-.651972	-.03931	.05552	95.099136	.436608
	15	-.051964	-.922049	-.034775	.056197	95.60862	.613499
	50	-.020443	-2.04959	-.012697	.058673	98.297272	1.258188
new kappas k=4,kP=8	-2	0	0	0	.0625	100	.05
	0	-.058072	-.023462	-.062916	.060378	92.855072	.053091
	3	-.048754	-.187868	-.050362	.059479	94.483627	.12399
	10	-.036739	-.489526	-.033239	.05926	96.553505	.327989
	15	-.031956	-.664363	-.026422	.059533	97.332184	.439036
	50	-.013932	-1.421665	-.008072	.061353	99.33799	.882921
new kappas k=4,kP=4	-2	0	0	0	.0625	100	.05
	0	-.042976	-.023237	-.048797	.06054	92.674995	.052977
	3	-.033094	-.141293	-.035541	.059466	94.309052	.093462
	10	-.02191	-.3346	-.019269	.059116	96.14772	.226744
	15	-.018563	-.438166	-.014068	.059302	96.672501	.295211
	50	-.00762	-.872255	-.004719	.061292	97.412209	.575431
new kappas k=8,kP=8	-2	0	0	0	.065217	100	.05
	0	-.067232	-.016709	-.096304	.067802	92.041847	.0505
	3	-.05201	-.203396	-.072649	.064979	95.118584	.138832
	10	-.033324	-.507937	-.040834	.06329	98.61412	.335544
	15	-.027891	-.669908	-.02971	.063114	99.706932	.429879
	50	-.014639	-1.407095	-.006112	.064305	101.82074	.8351

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